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Exploring complexity

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It has become a tradition of the Center for Statistical Mechanics and Thermodynamics to organize every year a workshop dealing with systems far from equilibrium. However, this year the workshop is held on a special occasion, as it is held in honour of Robert Herman, who was celebrating his 70th birthday in 1984.

Robert Herman is a man of many talents. To congratulate him on this occasion, we could have organized a workshop on cosmology or molecular physics, as he has obtained fundamental results in both fields. But it happens that he is best known among his colleagues for his contributions to operational research, specially for his work on the theory of vehicular traffic. This is an aspect of his work which I believe I am well acquainted with: We have been associated for many years, and we are the co-authors of a short monograph *Kinetic Theory of Vehicular Traffic*, which was published by Elsevier in 1971. This work was for us an example of what modelling complex systems may mean.

At first, vehicular traffic looks as quite difficult to be amenable to any quantitative treatment; however, this is not so. Introducing simple concepts, such as the desired velocity distribution of the drivers, one can easily set up non-linear differential equations, which express the competition between the wishes of the 'isolated driver' and the effect of the environment in which he is embedded.

We started this work in 1959. Robert Herman and his coworkers have pursued the program with great energy and perseverance, and—as you will undoubtedly hear from him—it has now reached a degree of perfection which makes it a useful tool to discriminate between the 'quality' of traffic in different towns. Even simple versions of these models lead to bifurcations, we could say to non-equilibrium phase transitions. In short, traffic theory presents many characteristics of non-linear systems.

I am sure this lesson was present to my mind when I was working on possible extensions of thermodynamics to far from equilibrium conditions, and pondering the possibility of the emergence of new, 'dissipative' structures.

As Robert Herman will discuss the problematics of traffic flow, I believe it will be more appropriate forme to present a general survey of some of the problems involved in the exploration of complexity.

1. In the classical perspective, there was a clearcut distinction between what was considered to be simple and what had to be considered as complex: There was no hesitation about calling 'simple' newtonian laws of motion, perfect gas, or chemical reactions. Also, one would have called 'complex' biological processes, and more so human activities such as described by economics or urban planning. In this perspective, the aim of classical science was to discover, even in complex systems, some underlying simple level. This level would be the carrier of deterministic (such carrier would be wave functions in the case of quantum mechanics) and time-reversible laws of nature: Future and Past would play the same role. However, this basic simple level remained elusive.

Today a far reaching reconceptualisation of science is going on. Wherever we look, we find evolution, diversification and instabilities. We long have known that we are living in a pluralistic world in which we find deterministic as well as stochastic phenomena, reversible as well as irreversible. We observe deterministic phenomena such as the frictionless pendulum or the trajectory of the moon around the earth, moreover, we know that the frictionless pendulum is also reversible.

But other processes are irreversible, as diffusion, or chemical reactions; and we are obliged to

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acknowledge the existence of stochastic processes if we want to avoid the paradox of referring the variety of natural phenomena to a program printed at the moment of the Big Bang. What has changed since the beginning of this century is our estimation of the relative importance of irreversibility versus reversibility, of stochasticity versus determinism [1].

Let us consider an example: the long-time variation of climate. We know that climate has fluctuated violently over the past. Climatic conditions that prevailed during the last two or three hundred millions years were extremely different from what they are at present. During this period, with the exception of the quaternary era (which began about two millions years ago) there was practically no ice on the continents, and the sea level was higher than its present value by about 80 meters. A striking feature of the quaternary era is the appearance of a series of glaciations, with an average periodicity of one hundred thousand years, to which is superposed an important amount of 'noise'. What is the source of these violent fluctuations which have obviously played an important role in our history? There is no indication that the intensity of the solar energy may have been responsible.

The temporal variation of climate is typically a 'complex process'. Again, in the perspective of classical physics, we would be tempted to attribute this complexity to a basic level, involving a large number of variables, which would enter into the determination of temperatures. The situation would then be similar to that induced by the law of 'large numbers', which leads to fluctuations distributed in a Gaussian manner.

Recent progress in the study of the behaviour of dynamical systems enable us to determine the number of independent variables linked through differential equations whose solution could generate the observed temporal sequence of temperature. The unexpected outcome of this analysis [2] is that the number of independent variables which determine the climate, is only 4. Therefore, we can no more ascribe the complexity observed to some underlying level, which would involve a large number of hidden variables. On the contrary, we have to attribute to the climate system an *intrinsic* complexity and *impredictability*.

In a quite different field, recent work [3] has shown that the electrical activity of the brain in deep sleep as monitored by electroencephalogram (EEG) may be modelled on a fractal attractor. Deep sleep EEG may be described by a dynamics involving 5 variables; again, this is very remarkable as it shows the brain acts as a system possessing intrinsic complexity and unpredictability.

It is this instability which permits the amplifications of inputs related to sensory impression in the awake state. Obviously, the dynamical complexity of the human brain cannot be an accident. It must have been selected for its very instability. Is biological evolution the history of dynamical instability, which would be the basic ingredient of creativity characteristic of human existence?

There have been other surprises. Even in some of the simplest examples of dynamics such as an elastic pendulum, unexpected complexity has been discovered [4,5], as it had been in some simple chemical reactions. It appears now that the gap between 'simple' and 'complex', between 'disorder' and 'order' is narrower than it was thought before.

Complexity is no longer limited to biology or human sciences: It is invading the physical sciences and appears as deeply rooted in the laws of nature.

These new developments are likely to be of decisive importance. For many scientists, the unknown was lying only at the frontiers of physics: in cosmology and in elementary particles. Today, the interest in macroscopic physics and chemistry, dealing with phenomena on our own scale, is rapidly increasing. Let me present three reasons which, I believe, explain this interest.

1. As we shall show, it leads to number of potentially innovative technical applications as well as to a better understanding of the main characteristics of our biosphere.
2. It gives us the possibility of transfer of the new theoretical tools coming from mathematical physics to biology and human sciences. It makes therefore the traditional distinction between hard and soft sciences obsolete.
3. The basic characteristics of complexity are irreversibility and stochasticity. These concepts begin now to diffuse into the fundamental level of description of nature.

2. One could state that the first science dealing with complexity in the field of physics and chemistry was the science of thermodynamics. The basic law of thermodynamics is the so-called Second Law, which expresses that entropy increases in isolated systems. (For more details, see [1,5].)

For a long time, the interest of thermodynamics concentrated on isolated systems at equilibrium. Today, interest has shifted to non-equilibrium, to systems interacting with their surroundings through an entropy flow. This interaction means that we are dealing with 'embedded' systems. This immediately brings us closer to objects like towns or living systems, which can only survive because of their embedding in their environment.

There is another basic difference with classical mechanics. Suppose we have some foreign celestial body approaching the earth: This would lead to a permanent change of the earth's trajectory: dynamical systems have no way to forget perturbations.

This is no longer the case when we include dissipation. A damped pendulum will reach a position of equilibrium, whatever the initial perturbation.

Now, when when we drive a system far from equilibrium, the 'attractor' which dominates the behavior of the system near equilibrium may become unstable, as a result of the flow of matter and energy which we direct at the system. Non-equilibrium becomes a source of order; new types of attractors, more complicated ones, may appear, and give a new space-time organisation to the system. Let us consider two examples which are widely studied today.

The so-called Bénard instability is a striking example of instability giving rise to spontaneous self-organisation; the instability is due to a vertical temperature gradient set up in a horizontal liquid layer. The lower face is maintained at a given temperature, higher than that of the upper. As a result of these boundary conditions, a permanent heat flux is set up, moving from bottom to top.

For small difference of temperature, heat can be conveyed by conduction, without any convection; but when the imposed temperature gradient reaches a threshold value, the stationary state (the fluid's state of 'rest') becomes unstable: convection arises, corresponding to the coherent motion of a huge number of molecules, increasing the rate of heat transfer. In appropriate conditions, the convection produces a complex spatial organisation in the system.

There is another way of looking at this phenomenon. There are two elements involved: heat flow and gravitation. Under equilibrium conditions, the force of gravitation has hardly any effects on a thin layer of the order of 10 mm. In contrast, far from equilibrium, gravitation gives rise to macroscopic structures.

Non-equilibrium matter is much more sensitive to its environment than matter at equilibrium. I like to say that at equilibrium, matter is blind; far from equilibrium it may begin to 'see'.

Consider next chemical oscillations. We study a chemical reaction whose state we control through the appropriate injection of chemical products and the elimination of waste products. Suppose that two of the components are formed respectively by red and blue molecules in comparable quantities. We would expect to observe some kind of blurred color with perhaps occasionally some flash of red or blue spots. This is, however, not what actually happens. For a whole class of such chemical reactions, we see in sequence the whole vessel become red, then blue, then red again: We have a 'chemical clock'. This violates our intuition about chemical reactions. [6]

We were used to speak of chemical reactions as being produced by molecules moving in a disordered fashion and colliding at random. But, in order to synchronize their periodic change, the molecules must be able to 'communicate'. In other words, we are dealing here with new supermolecular scales—both in time and space—produced by chemical activity.

The basic conditions to be satisfied for such chemical oscillations to occur are auto- or cross-catalytic relations, leading to 'non-linear' behaviour, such as described in numerous studies of modern biochemistry. Remember that nucleic acids produce proteins, which in turn lead to the formation of nucleic acids. There is an autocatalytic loop involving proteins and nucleic acids.

Non-linearity and far-from equilibrium situations are closely related; their effect is that they lead to a multiplicity of stable states (in contrast to near-from-equilibrium situations, where we find only one stable state).

This multiplicity is to be seen on a 'bifurcation diagram' if we plot the solution of some nonlinear problem against a bifurcation parameter (for example the concentration of some chemical component versus the time of sojourn of the molecules in a chemical reactor). For some critical value of this time, new solutions emerge. Moreover, near the bifurcation point, the system has a 'choice' between two branches: We therefore expect fluctuations to play an essential role.

We mentioned the fact that dissipative systems may forget perturbations: These systems are characterized by attractors. The most elementary attractors are points or lines. But attractors may present a more complex structure; they may be formed of a set of points. Their distribution may be dense enough to permit us to ascribe them a fractal dimensionality [7].

Such systems have unique properties, reminiscent of turbulence which we encounter in everyday experience. They combine both fluctuations and stability. The system is driven to the attractor still, as this one is formed by so 'many' points, we may expect large fluctuations. One often speaks of 'attracting chaos'. These large fluctuations are connected to a great sensitivity in respect to initial conditions. The distance between neighbouring trajectories grows exponentially in time. Attracting chaos has now been observed in a series of situations including chemical systems or hydrodynamics; but the importance of these new concepts goes far beyond physics and chemistry properly. We have already indicated the examples of long-term behaviour of climate or the electrical activity of the brain; there is no doubt that the new concepts are essential features of our environment; their study will permit to model complex behaviour displayed by systems in ecology or economics.

3. The physics and chemistry of complex phenomena leads at present to a new interface between 'pure' and 'applied' research. This interface is growing so rapidly that I can only briefly enumerate a few examples.

A characteristic feature of far-from-equilibrium conditions is the possibility of bistability. For given boundary conditions, there may be more than one stable solution [5].

A remarkable application of bistability is in optonics, in which the intensity of a coherent light beam through a resonant cavity may induce more than one stable value of the transmitted intensity [8]. This bistability appears as a transposition to optics of the hysteresis phenomenon well known in magnetism.

The stable states of the system are a function of its history, and not only of the boundary conditions: For a given value of the incident light intensity, it will evolve toward the low transmission branch (opaque state) if it enters the bistable zone coming from below or toward the high transmission branch (transparent state) if it comes from above. It therefore acts as a binary memory.

Potential advantages of optical memories are: three orders of magnitude as far as speed of response is concerned (from 10^{-9} to 10^{-12} seconds); and parallel processing, as a bistable optical element whose section is 1 cm^2 may easily process in parallel more than 10^3 informations. What is perhaps more important: These components are susceptible to act as optical transistors.

It is interesting that this phenomenon of bistability is present in many problem, for example in biological cell dynamics. A simple example, which has been studied by my colleagues in Brussels, is the interaction between tumor cells and immune system cells which kill tumor cells [9]. Most of the effort in the study of cancer is directed to discover the mechanisms which lead to the transformation of a cell to a cancerous cell. In contrast, here we concentrate on the response of the organism to a given population of cancer cells. Basically, a minimal dynamical model would be one in which cancer cells form complexes with cytotoxic cells, which are then regenerated after having killed cancer cells. This situation may lead to one or many steady states. One observes that each cytotoxic cell may bind more than one tumor cell; this leads to highly non-linear processes; for this reason, one has to expect multiple states. In this perspective, one of the main approaches of cancer would be to study the transition from a dormant form of cancer to a virulent one.

Other recent research concerns nucleation of fractures and the initiation of plasticity in materials submitted to stress [10]. As well known, every material contains defects. Under stress, some immobile dislocation may become mobile and interact. There is then an obvious analogy with the reaction/diffusion equations, which have been widely studied for chemical systems far from equilibrium.

In conditions involving stress, there is a possibility of spatial dislocation patterns, leading to an accumulation of dislocations in some regions. These regions, which have been observed experimentally, are then likely to lead to fractures and plasticity [11].

I would like also to mention two types of problems which, in addition to non-linearity, involve fluctuations. Ideally speaking, for systems presenting a bifurcation point leading from one stable solution to two stables solutions, the probabilities of selecting one branch against the other are equal. But completely symmetrical solutions are only limit cases. Currently, we deal with '*imperfect*' bifurcations, which can play a crucial role in the selection of the outcome. An extreme example is the selection of chiral molecules, in which a very small difference in the energy of formation of the molecule could lead to preferential selection. This is basically due to the possibility of polarizing the fluctuations near the bifurcation point [12].

We begin to understand also other cases, some of which have great potential importance, such as combustions and ignition where the deterministic description breaks down [13]. We have an initial induction regime, characterized by a very small rate of change which is followed by a violent explosive behaviour. As the result of the induction stage, fluctuations play an important role: One finds a statistical distribution of ignition times instead of a simple, deterministic ignition time.

Abnormal fluctuations have also been observed in many biological problems, such as the distribution of growth rate of young males and females near puberty. This also indicates the existence of an autocatalytic effect, with a long induction period, as is the case in combustion. It would be fascinating to examine these ideas in learning processes, which often proceed by steps as has been indicated by Piaget, and are likely to have long inductive periods.

4. The discovery of the constructive role of irreversible processes in physics and chemistry and of their importance in understanding physical processes as well as the behaviour of the biosphere leads us to reconsider the microscopic meaning of irreversibility. Traditionally, irreversibility was only tolerated on the macroscopic level. It was supposed to be the result of our ignorance of the exact dynamical state of the system. In contrast, on the basic microscopic level, there would be no question of an arrow of time, and no irreversibility.

This problem is closely related to the transition from the basic description involved in classical or quantum mechanics, with its deterministic and time-reversible features, to a description in which probability and irreversibility play a fundamental role. Only a few years ago, this problem seemed to be impossible to solve. The two descriptions, the dynamical one and the thermodynamical one, seemed to be separated by a gap which could not be bridged.

We begin now to see a way out of this difficulty. I would like to describe briefly the basic ideas involved. Classical mechanics may be seen as a point transformation in the phase space formed by coordinate and moment. Another way of looking at dynamical evolution is in terms of a set of points which occupy some volume zone in phase space. A characteristic feature of classical mechanics is that this volume (the 'measure', to use the mathematical term) is conserved in time.

This does not exclude very complex situations. The volume may be highly deformed or even broken into small pieces. This destruction of the initial 'simple' volume gives the appearance of an approach to equilibrium distribution in the phase space.

Classical physics shows that conservation of volume and conservation of information are closely related. That is the reason why, in classical dynamics, information is strictly conserved. Initial conditions can be restituted. Indeed, the fragments of the initial volume could be brought back simply by inverting the direction of time.

We see how different the world appears in the thermodynamical description. For the mechanical description, the world appears as a museum in which everything, including information, is conserved. The world of thermodynamics is a world of processes, destroying and creating information; the volume is no more conserved. Think of the evolution of temperature, the inhomogeneity of which disappears without leaving any trace.

The new feature is that for a well defined class of dynamical systems, we may now go from one description to another. This class is precisely the one in which the initial volume is highly deformed and broken into pieces, time going on. Such systems are highly unstable from the dynamical point of view. Moreover, in such systems, not all initial conditions should be possible. Only initial conditions leading to equilibrium *in the future* are actually susceptible of realisation [1,14].

We begin therefore to be able to spell out the basic message of the second law. This message is that we are living in a world of unstable dynamical systems:

If the world were build at the image designed for reversible, eternal systems by Galileo Galilei and Isaac Newton, there would be no room for irreversible phenomena such as chemical reactions or biological processes.

For unstable systems, which have a privileged time direction, we see a dispersion of the initial volume in phase space. Then, we cannot impose initial conditions which would force an ensemble of points to concentrate on a single point. The future remains open.

The message carried by the second law is therefore not one of ignorance and subjectivity. On the contrary, it gives us some basic information about the overall structure of the physical world.

At the beginning of this lecture, we referred to a basic level of physical description. We have now to take into account the second law of thermodynamics, even on this level. Therefore, this level can be formed neither by trajectories nor by wave functions, which satisfy deterministic equations in which the Future would be already included in the Present.

Whenever thermodynamics is valid, the basic objects of physics must be objects less specified than trajectories or wave functions. The new objects are then driven, as time goes on, to equilibrium in closed systems—or, in the presence of appropriate conditions, to 'dissipative structures'. But we cannot go further into this fascinating subject.

5. Let us summarize our main findings. The universe has a history. This history includes the creation of complexity through mechanisms of bifurcation. These mechanisms act in far from equilibrium conditions as realised in the earth's biosphere. They may also have been of special relevance in the early stage of the universe, where we have to expect a strong coupling between matter and gravitation [15].

Non-equilibrium physics is at present a subject in a state of explosive growth. I have tried to show you in this lecture some of the reasons for this fascination. It leads both to new applications of direct scientific and technical importance, and to new perspectives on the very foundations of physics, which will also be likely to lead to new technology developments in the next century.

Rationality can no longer be identified with 'certainty', nor probability with ignorance, as has been the case in classical science. At all levels, in physics, in biology [16], in human behaviour [17], probability and irreversibility play an essential role. We are witnessing a new convergence between two 'visions of the world', the one emerging out of scientific experience, and the other we get from our personal existence, be it through introspection or through existential experience.

Sigmund Freud told us that the history of science is the history of an alienation: Since Copernicus we no longer live at the centre of the universe; since Darwin, man is no longer different from other animals; and since Freud himself conscience is just the emerged part of a complex reality hidden from us.

Curiously, we now reach the opposite view. With the role of duration and freedom so prevalent in human life, human existence appears to us as the most striking realization of the basic laws of nature.

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