

Planar photonic bandgap structures

(PhD course: Optical at the nanoscale)

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Outline

- Introduction to photonic bandgap structures
- The plane-wave-expansion method (MIT method)
- 2D photonic crystal waveguides
- Planar slab photonic crystal waveguides
- Large-bandwidth planar photonic crystal waveguides
- Photonic crystal micro-cavities

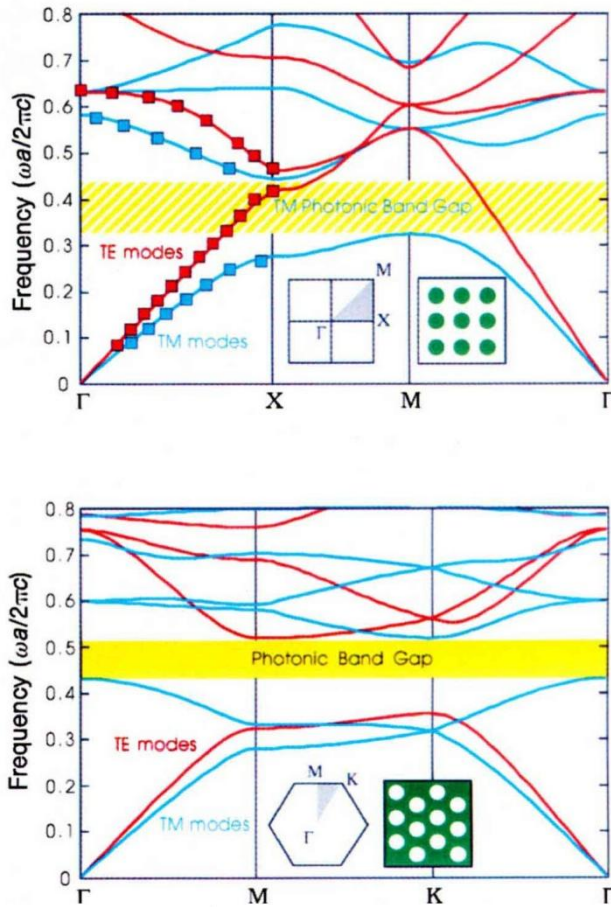


Figure 1 Top, photonic band structure for a square lattice of dielectric ($\epsilon = 8.9$) rods in air with radius $r = 0.2a$, where a is the lattice constant. TM modes are shown in blue and TE modes in red. The solid lines are from theory and the squares represent experimental measurements along Γ to X from Robertson *et al.*⁹ Bottom, photonic band structure for a triangular lattice of air cylinders ($r = 0.48a$) in dielectric ($\epsilon = 13$). Note the presence of a complete photonic bandgap for both TE and TM polarizations in this case as shown by a solid yellow bar. In both panels high-dielectric material is indicated in green in the insets.

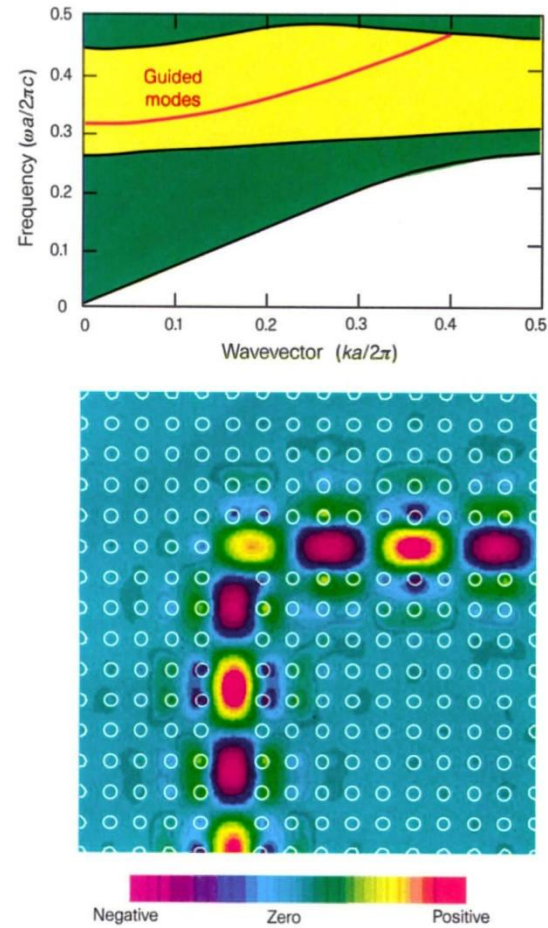


Figure 2 Top, projected photon bands for a waveguide in a square lattice of dielectric rods. The waveguide is formed by removing one row of rods from the otherwise perfect lattice and this creates a band of guided modes as shown in red. The green regions correspond to propagating modes in the bulk crystal and the yellow region corresponds to the photonic bandgap. Bottom, electric field of light propagating down a waveguide with a sharp bend carved out of a square lattice of dielectric rods. The white circles indicate the positions of the rods.

review article

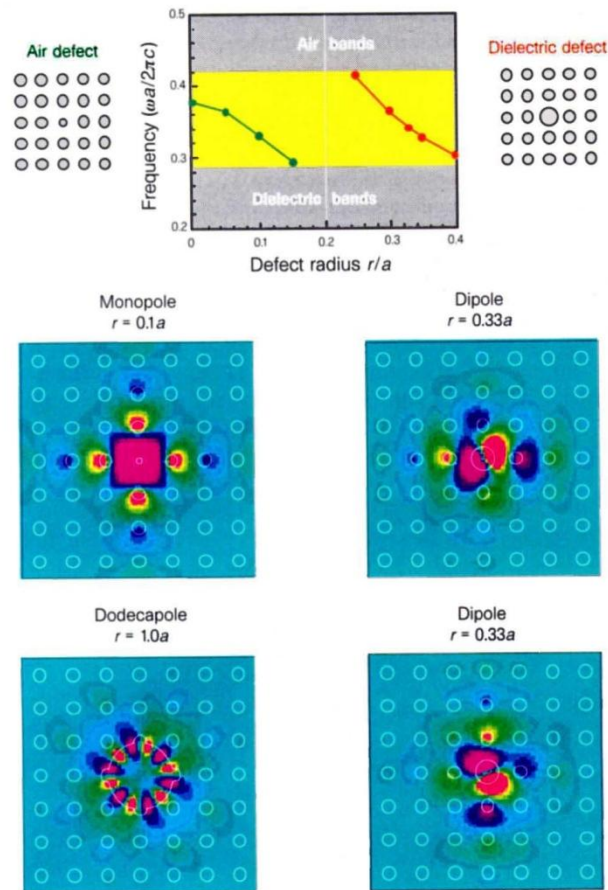


Figure 3 Top, localized states in the gap for a defect formed by varying the radius of a single rod in the square lattice of dielectric rods with lattice constant a . The case where there is no defect corresponds to $r = 0.2a$, while the case of a vacanc corresponds to $r = 0$. Bottom panels, electric field patterns associated with selected defect states as indicated. The colour coding is the same as in the bottom panel of Fig. 2.

The plane-wave-expansion method

We choose to work with the wave equation for the magnetic field because it is a simple eigenvalue equation, and it is *Hermitian* for dielectric materials without absorption.

$$\nabla \times \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{r}) \quad \begin{array}{l} \text{Eigenvector: } \mathbf{H}(\mathbf{r}), \\ \text{Eigenvalue: } \omega^2/c^2 \end{array}$$

Field expansion for structures with discrete translational symmetry:

$$\mathbf{H}_{\mathbf{k},n}(\mathbf{r}) = \mathbf{U}_{\mathbf{k},n}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_{\mathbf{G}} \sum_{\lambda=1,2} h_{\mathbf{k},\mathbf{G},\lambda,n} e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}}$$

When the following functional is at a minimum, \mathbf{H} is an eigenvector and $E(\mathbf{H})$ is the corresponding eigenvalue. The expansion coefficients are adjusted using an iterative procedure until the minimum is reached.

$$E(\mathbf{H}) = \frac{\left\langle \nabla \times \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H} \middle| \mathbf{H} \right\rangle}{\langle \mathbf{H} | \mathbf{H} \rangle}$$

Interpretation of the slope of dispersion curves (2D)

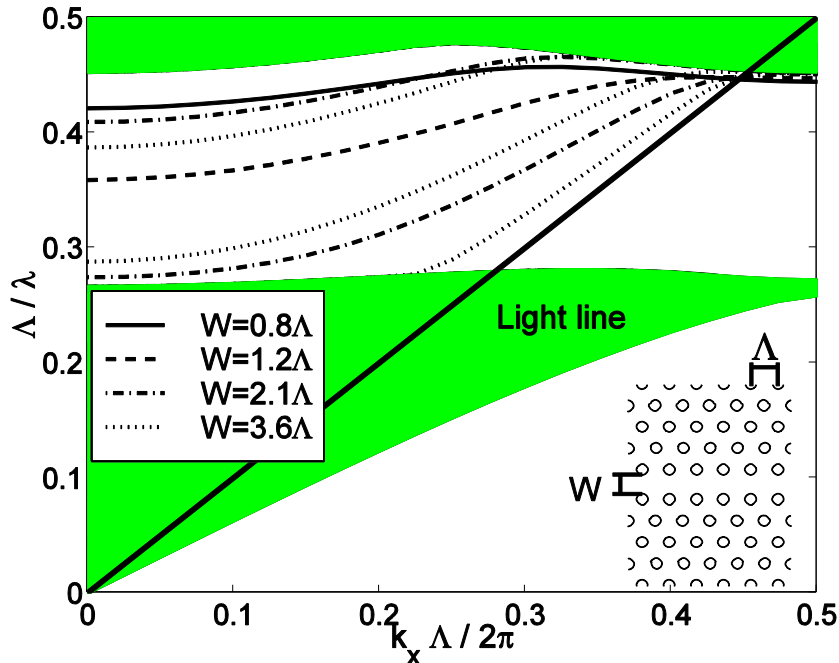


Fig. 1

2D PCW with silicon rods surrounded by air.

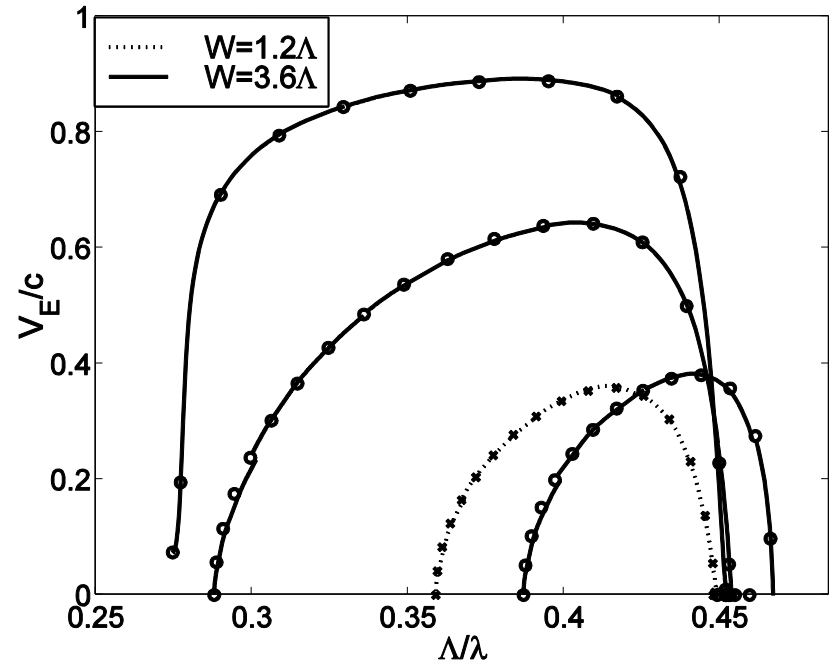


Fig. 2

Energy propagation velocity

$$V_E = \frac{\int_{y=-\infty}^{y=+\infty} P_x(\mathbf{r}) dy}{\frac{1}{\Lambda} \int_{y=-\infty}^{y=+\infty} \int_{x=0}^{x=\Lambda} \left(\epsilon_0 \epsilon_r(\mathbf{r}) |\mathbf{E}(\mathbf{r})|^2 + \mu_0 |\mathbf{H}(\mathbf{r})|^2 \right) dx dy}$$

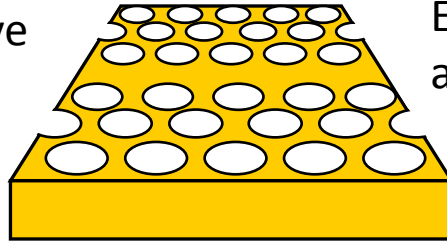
- The smooth curves in Fig. 2 correspond to the slope of the dispersion curves in Fig. 1.
- The discrete points in Fig. 2 is the energy propagation velocity calculated from the Poynting vector and the energy stored in the fields.
- Note that the curves approach the light line in Fig. 1 as W is increased.

Reference: Phys. Rev. B **61**, 15688 (2000).

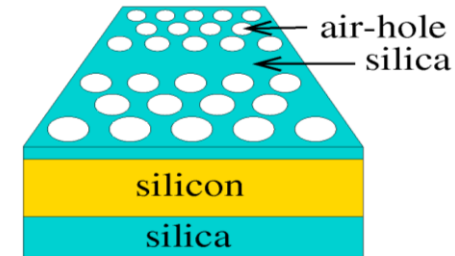
Considerations regarding orientation and period of planar photonic crystal waveguides

· A slab of a 2D PCW will usually be surrounded by a homogeneous dielectric material above and below the slab such as air or glass:

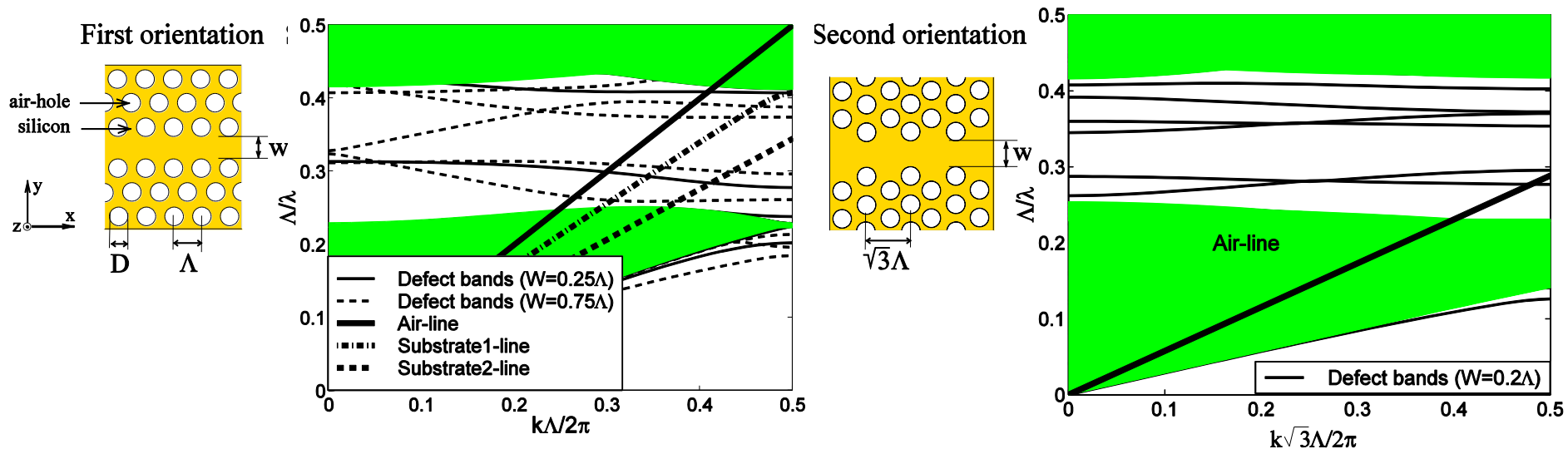
Example with air above and below the slab



Example with glass above and below the slab



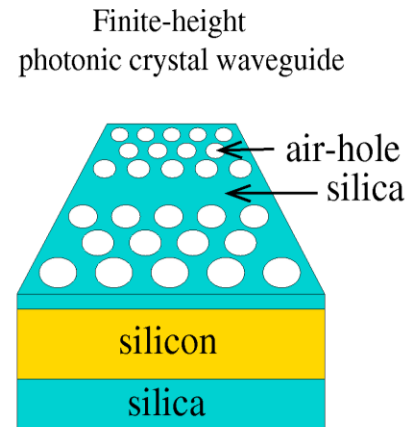
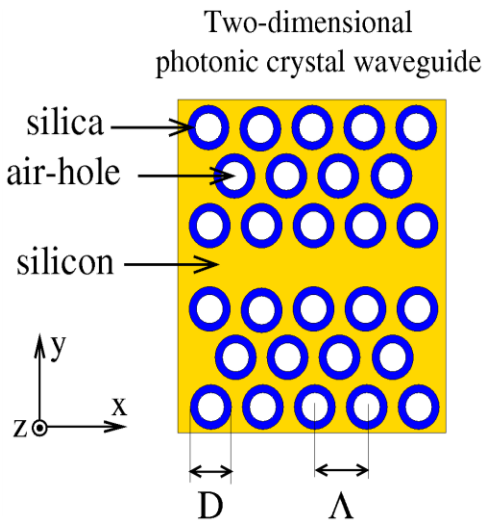
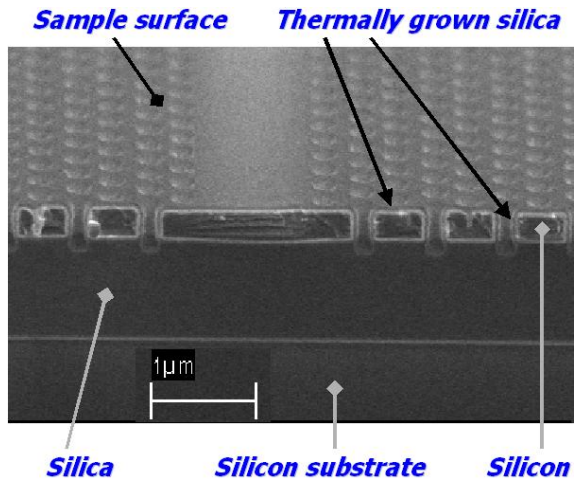
· If a combination of Bloch wave number and frequency is allowed in the media surrounding the 2D PCW slab a mode with these characteristics will leak energy into the surrounding media



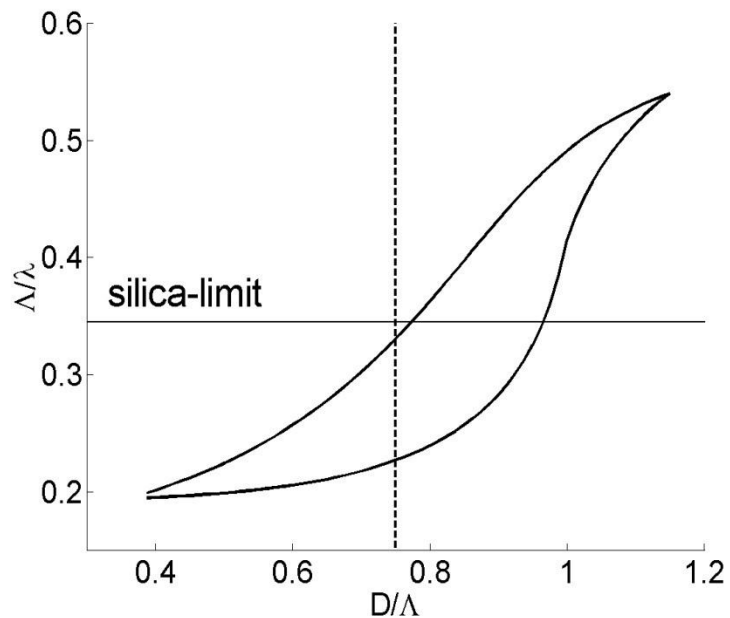
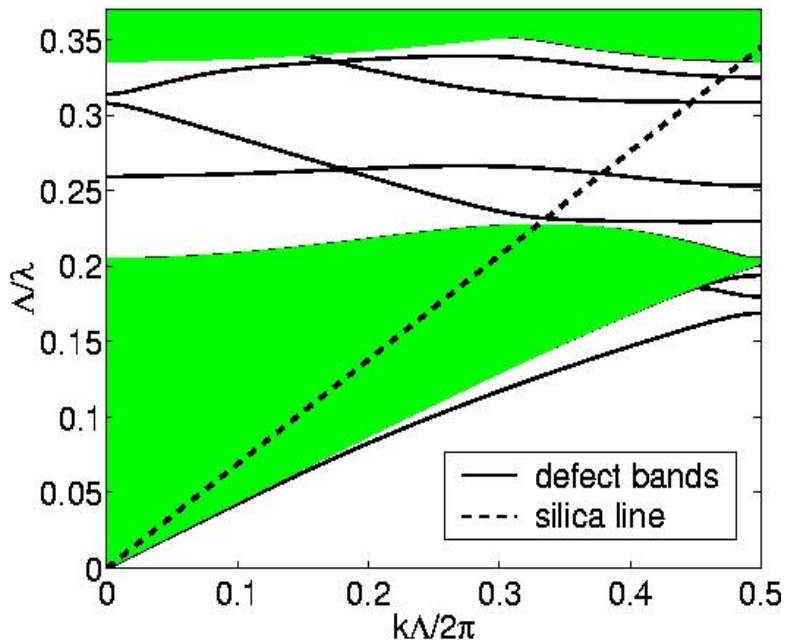
- The surrounding medium should have low refractive index
- The period of the waveguide should be small - "First orientation" is preferable

Reference: Appl. Phys. Lett. **77**, 785 (2000).

Example of a silicon-on-insulator (SOI) PCW (2D)

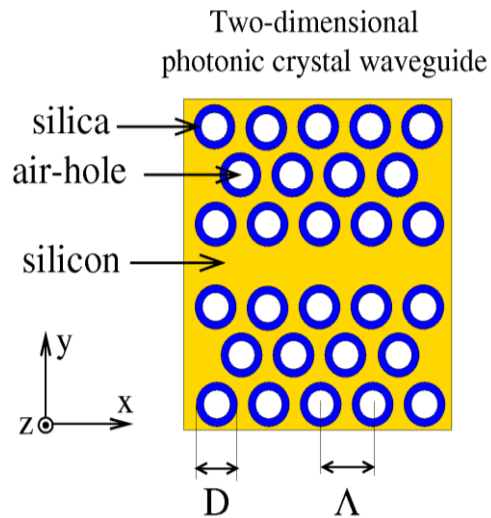
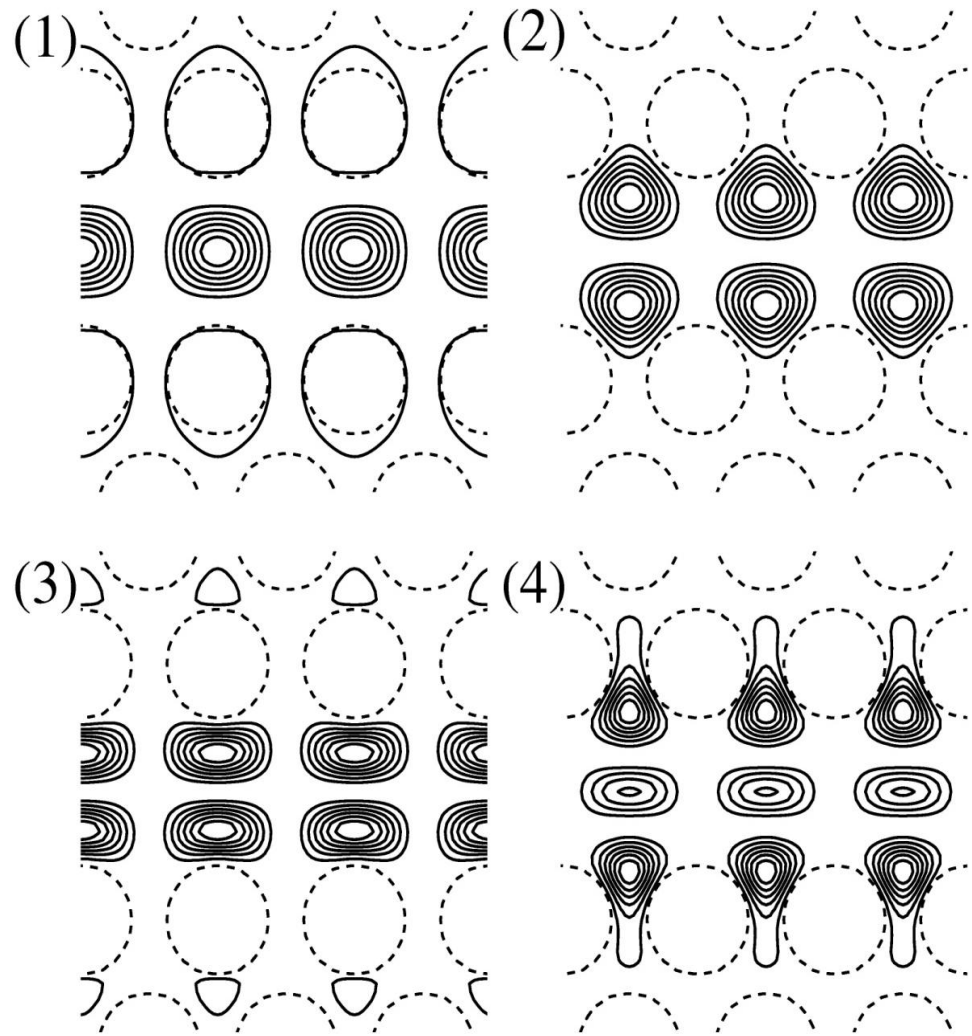
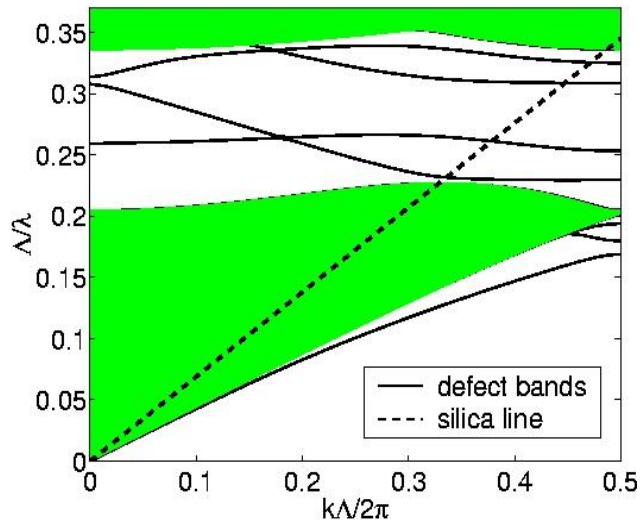


Dispersion relations (2D)



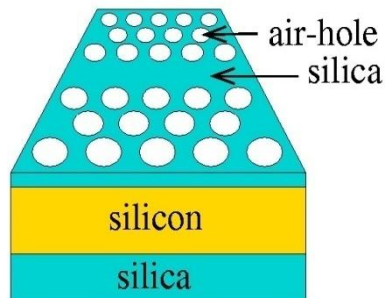
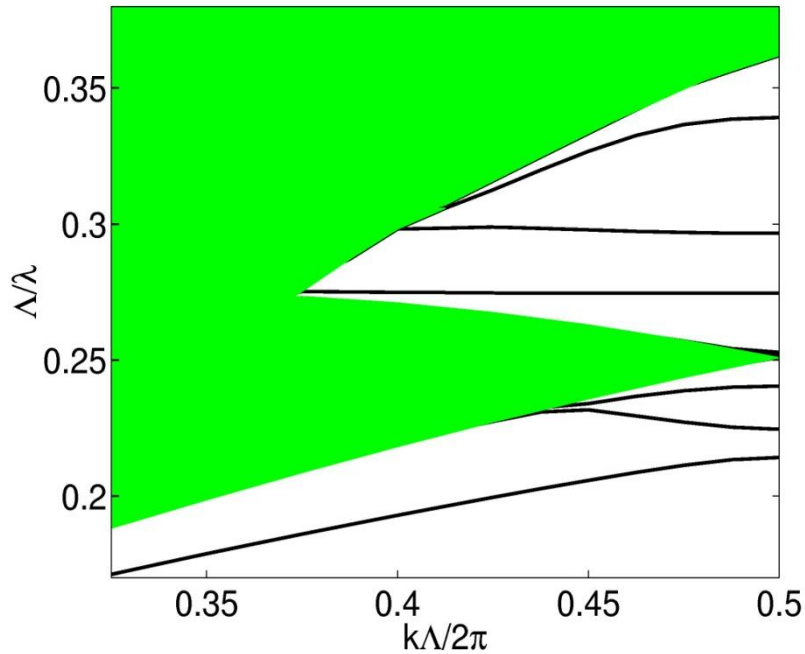
Reference: J. Lightwave Technol. **20**, 1619 (2002).

Amplitude of magnetic field squared for bandgap guided modes in a two-dimensional photonic crystal waveguide

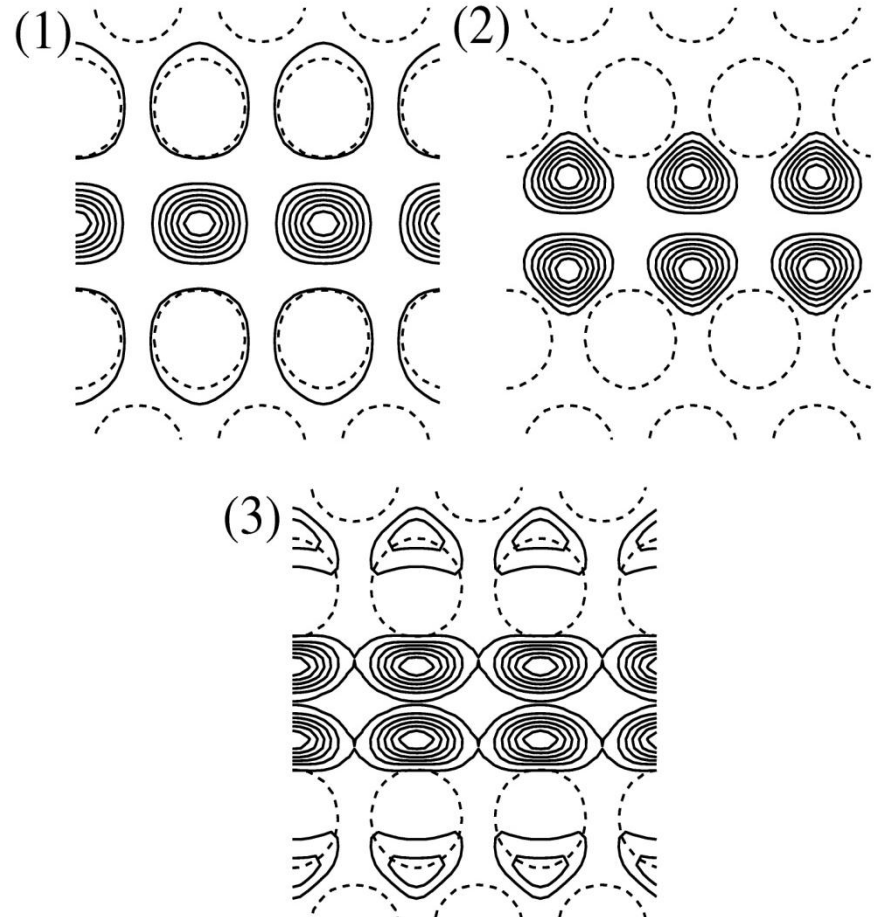


Example of a silicon-on-insulator (SOI) PCW (3D)

Dispersion relations (3D)

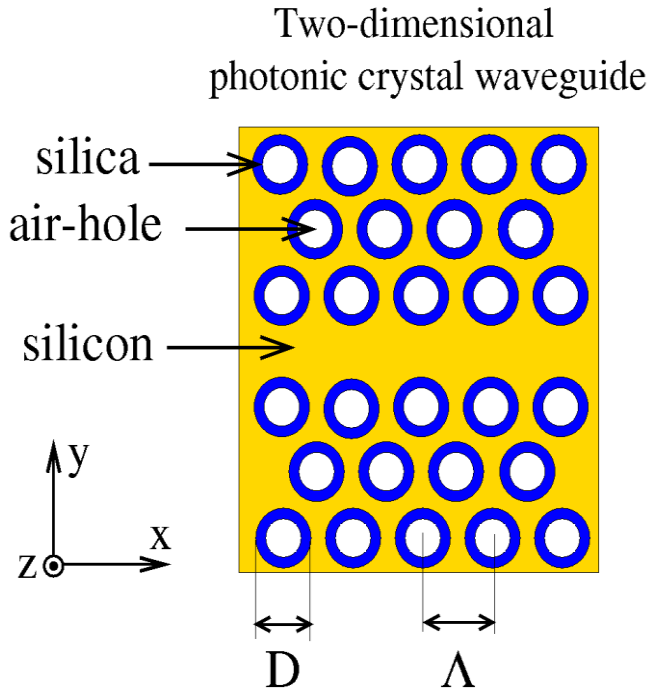


Amplitude of magnetic field squared for the bandgap guided modes

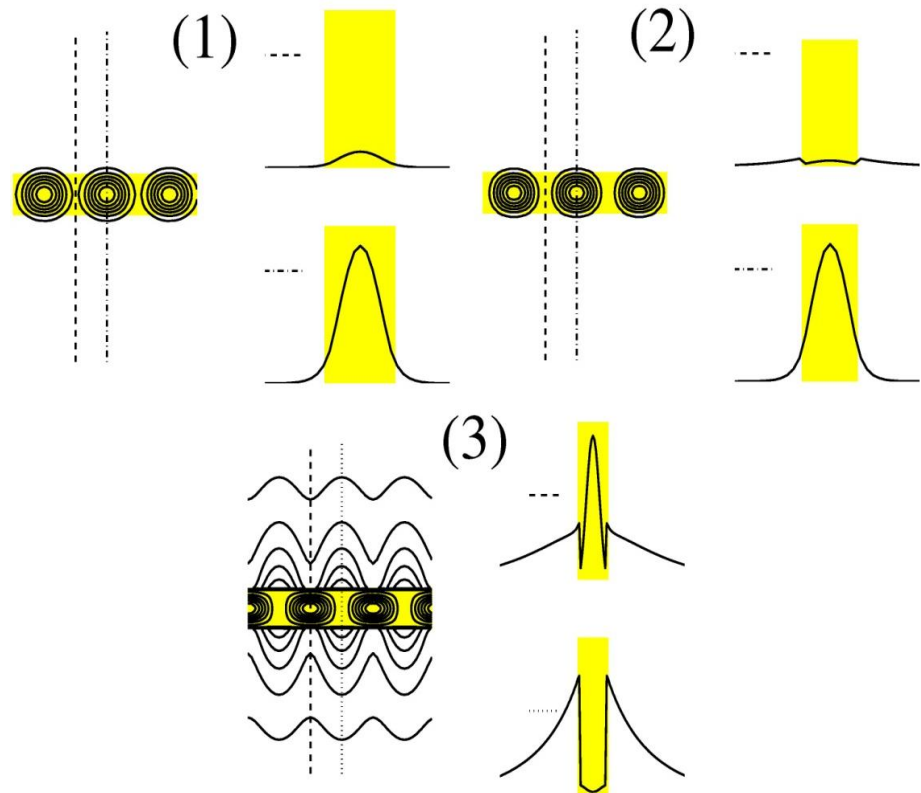


Three-dimensional analysis of finite-height photonic crystal waveguides

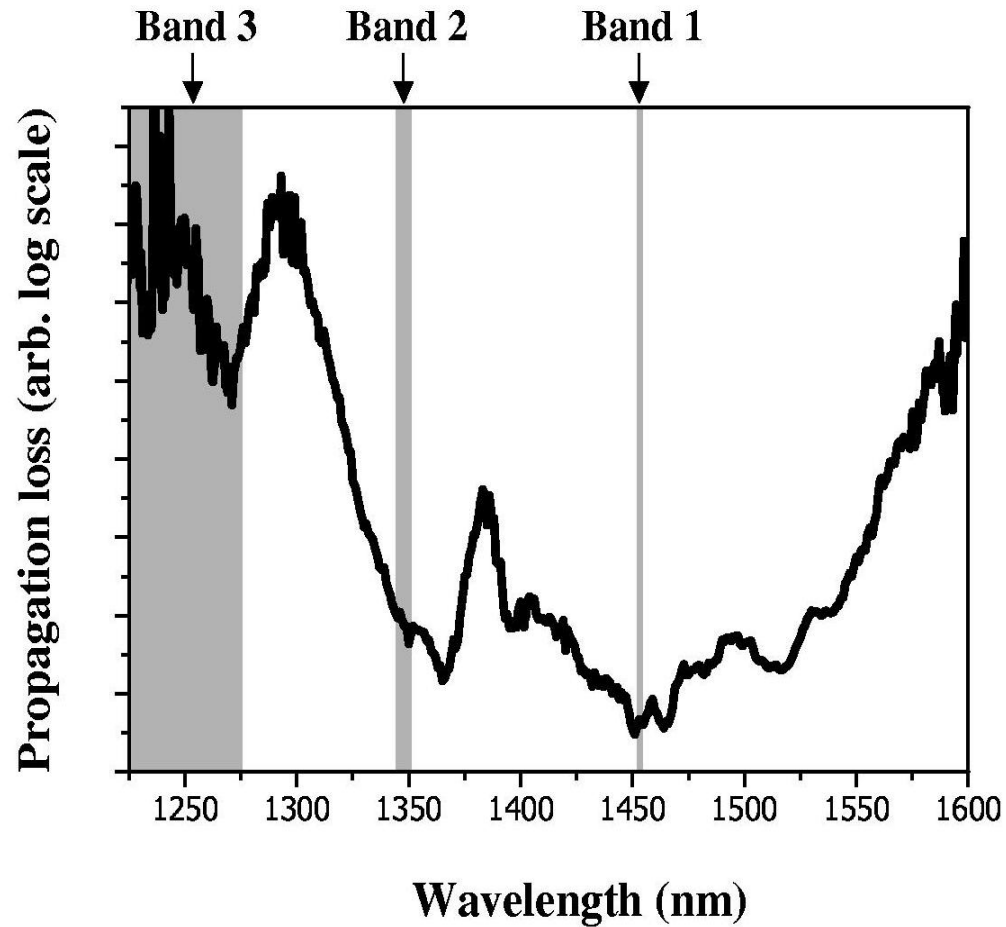
Amplitude of electric field squared for the three bandgap guides modes ($k\Lambda/2\pi=0.46$) for a plane defined by the waveguide axis and the z-axis



Note the long tails of the electric field for mode (3)

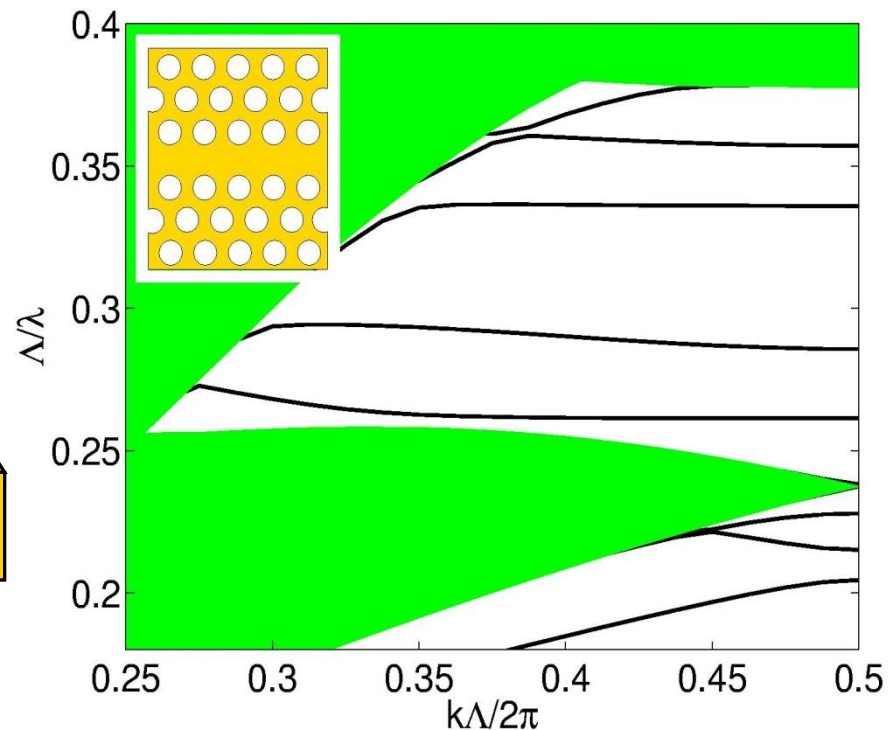
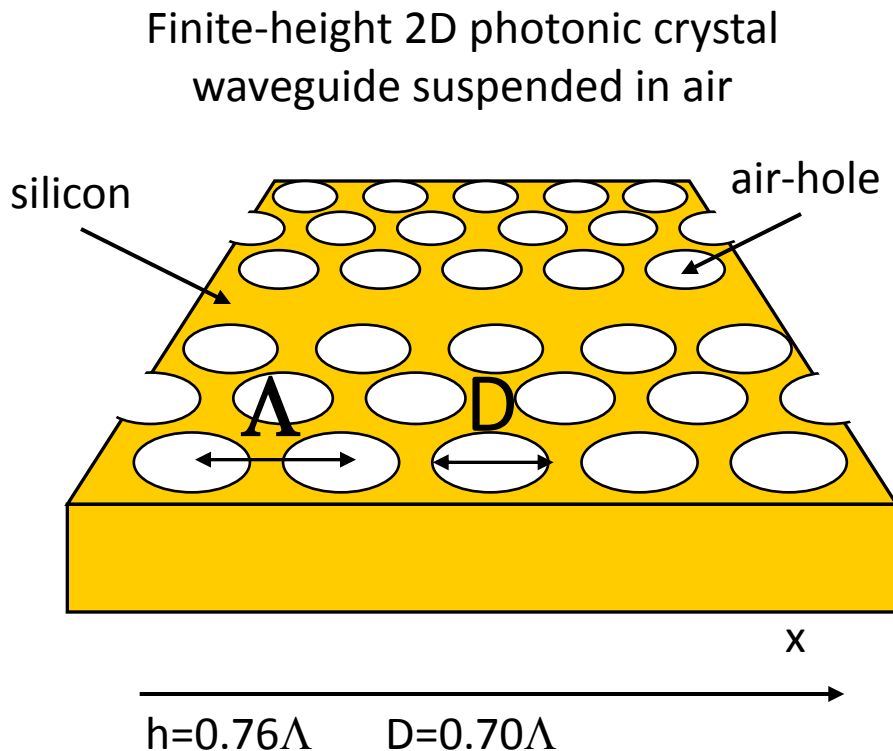


Comparison with experiment



The measurement has been made by J. Arentoft.

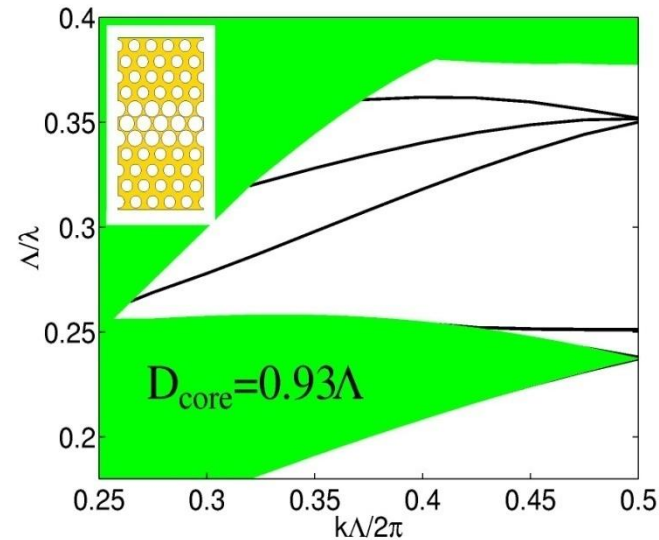
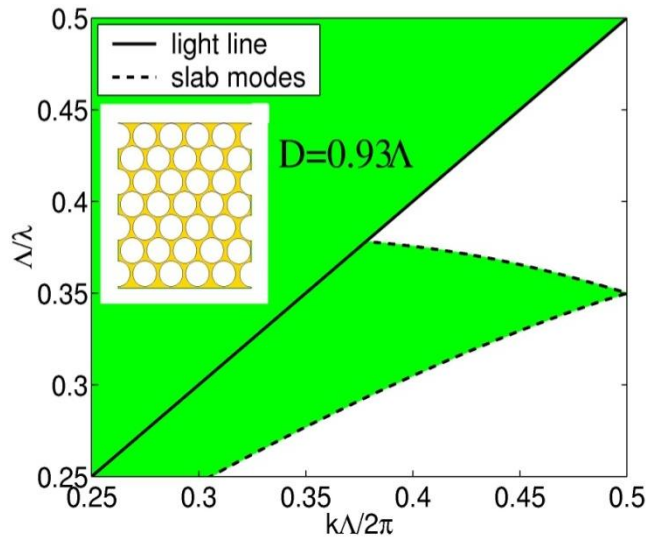
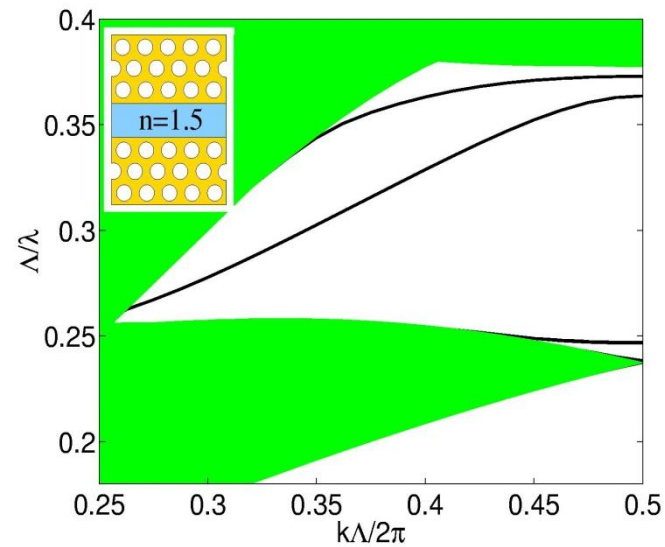
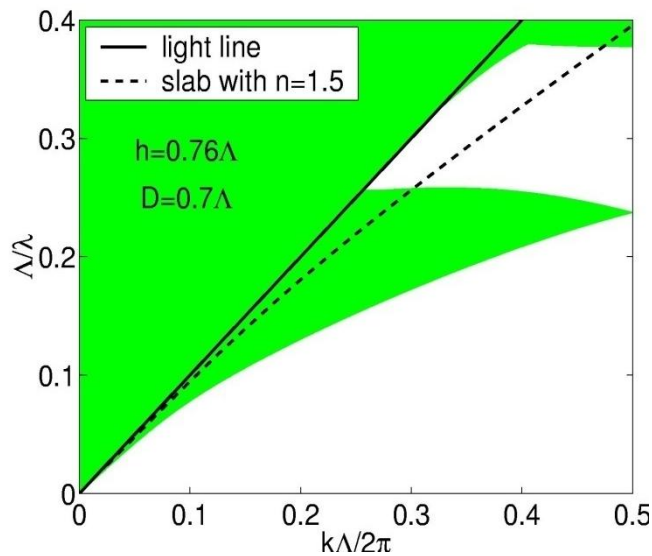
Design principle for achieving a large bandwidth



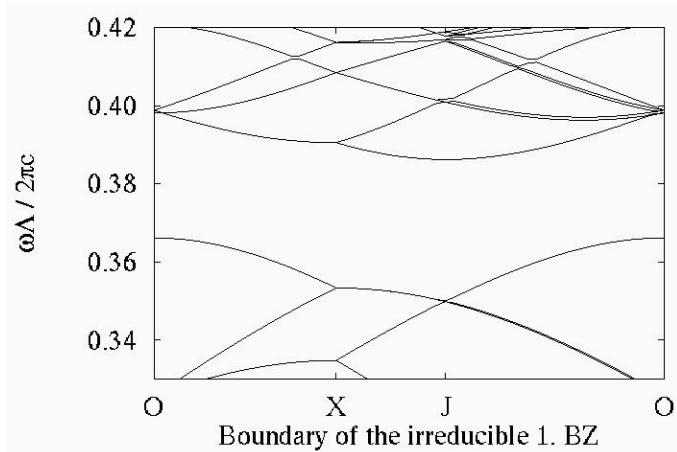
- The discrete bands cover only narrow frequency intervals => the waveguides are narrow-bandwidth waveguides
- The bands are flat => high group-index / low energy propagation velocity

Design principle for achieving a large bandwidth

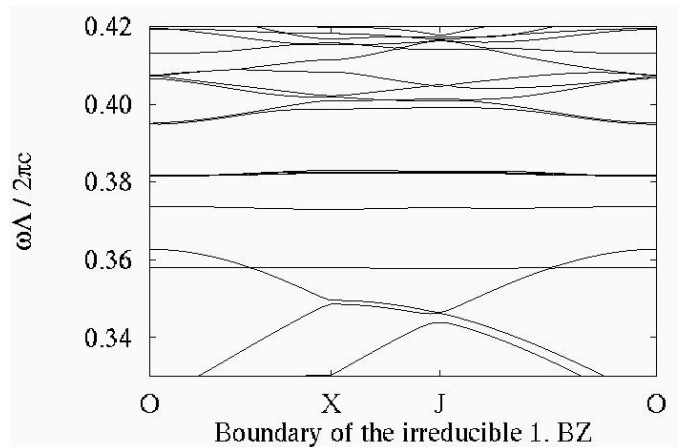
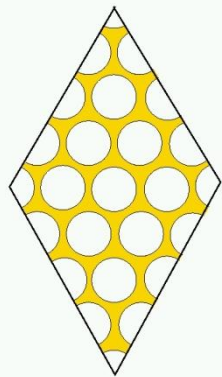
Idea: The dispersion curves of guided modes will approach the dispersion curves of a slab of the material in the line-defect when the line-defect width becomes large



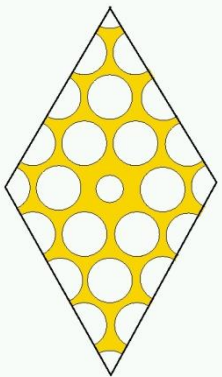
Reference: Optics Communications **203**, 263-70 (2002).



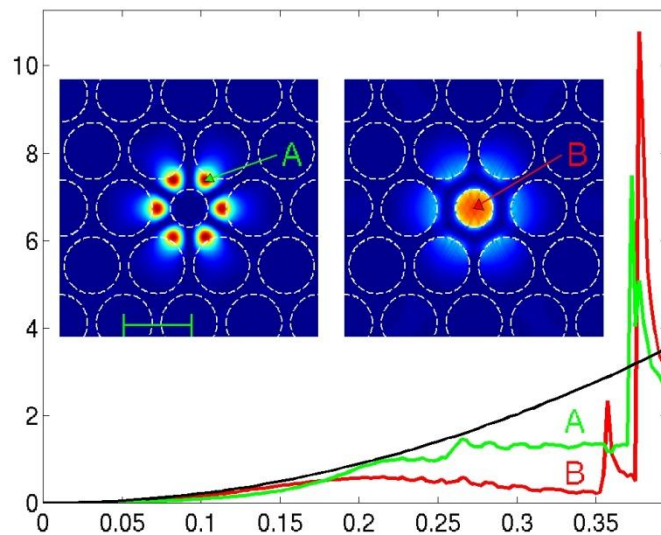
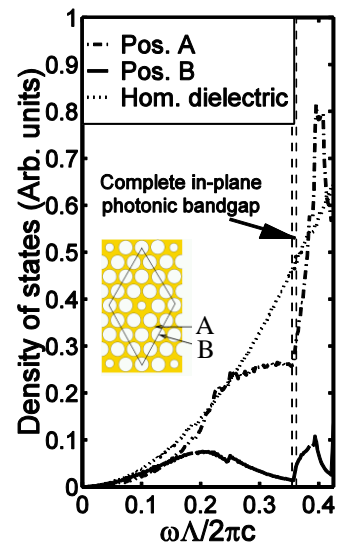
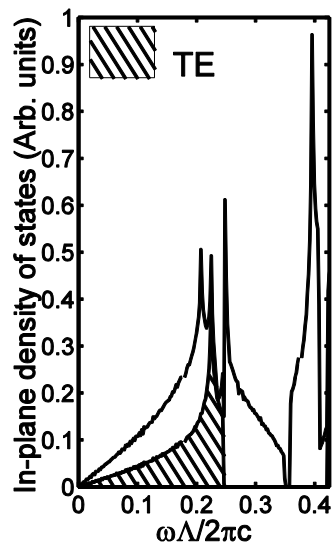
Super cell



Super cell



2D / 3D density of states



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