Photonic crystal fibres

(PhD course: Optical at the nanoscale)

Thomas Søndergaard

Department of Physics and Nanotechnology
Aalborg University, Denmark
Outline:

Brief review of the standard step index fiber

Index guiding PCF’s
- Endlessly single-mode fibers / large core fibers

Bandgap guiding PCF’s
- Guiding light in air-holes

Example of data sheet
The standard step-index fiber

\[ n = \begin{cases} n_1 & \text{for } \rho \leq a, \\ n_2 & \text{for } \rho > a. \end{cases} \]

Bølgeligningen for det elektriske (eller magnetiske) felts z-komposant:

\[
(\nabla^2 + k_0^2 n^2)E_z = \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + k_0^2 n^2 E_z = 0
\]

\[ E_z(\rho, \phi, z) = F(\rho)\Phi(\phi)Z(z) \]

\[
\frac{d^2Z}{dz^2} + \beta^2 Z = 0, \quad \frac{d^2\Phi}{d\phi^2} + m^2 \Phi = 0, \quad \frac{d^2F}{d\rho^2} + \frac{1}{\rho} \frac{dF}{d\rho} + \left( k_0^2 n^2 - \beta^2 - \frac{m^2}{\rho^2} \right) F = 0
\]

\[ E_z = \begin{cases} AJ_m(\kappa \rho) e^{i m \phi} e^{-i \beta z} & \text{for } \rho \leq a \\ CK_m(\gamma \rho) e^{i m \phi} e^{-i \beta z} & \text{for } \rho > a \end{cases} \]

\[ H_z = \begin{cases} BJ_m(\kappa \rho) e^{i m \phi} e^{-i \beta z} & \text{for } \rho \leq a \\ DK_m(\gamma \rho) e^{i m \phi} e^{-i \beta z} & \text{for } \rho > a \end{cases} \]

\[ \frac{E_\rho}{\kappa^2} i \left( \beta \frac{\partial E_z}{\partial \rho} + \mu_0 \omega \frac{\partial H_z}{\partial \phi} \right), \quad E_\phi = i \kappa^2 \left( \beta \frac{\partial E_z}{\rho \partial \phi} - \mu_0 \omega \frac{\partial H_z}{\partial \rho} \right) \text{ for } \rho \leq a \]

\[ \frac{H_\rho}{\kappa^2} i \left( \beta \frac{\partial H_z}{\partial \rho} - \epsilon_0 n_1^2 \omega \frac{\partial E_z}{\rho \partial \phi} \right), \quad H_\phi = i \kappa^2 \left( \beta \frac{\partial H_z}{\partial \phi} + \epsilon_0 n_1^2 \omega \frac{\partial E_z}{\partial \rho} \right) \]
The standard step-index fiber

\[ V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2} \]

A standard step-index fiber becomes multi-mode when the V-parameter exceeds 2.405.

The number of modes that a fiber supports depends on
1) how strong the contrast is between core and cladding
2) the size of the core region (relative to the wavelength)

The theory of the standard step-index fiber can e.g. be found in:
G.P. Agrawal, ”Fiber-optic communication systems”, Wiley & Sons, New York, 1992
The "index" of the cladding increases with increasing frequency for a PCF.

\[ V_{\text{eff}} = \frac{2\pi}{\Lambda} \Lambda \sqrt{n_{\text{core}}^2 - n_{\text{eff,cladding}}^2} \]

\[ n_{\text{eff,cladding}} = \beta_{\text{FSM}} / k_0 \]


Fig. 3. Variation of \( V_{\text{eff}} \) with \( \Lambda/\lambda \) for various relative hole diameters \( d/\Lambda \). The dashed line marks \( V_{\text{eff}} = 2.405 \), the cutoff \( V \) value for a step-index fiber.
Super cell approximation method

The plane wave method requires a periodic structures.

The cladding material of a photonic crystal fibre is periodic.

However, a photonic crystal fibre is not a periodic structure due to the core region. This problem is overcome by approximating the structure with a periodic structure in which the periodically repeated core regions are separated sufficiently far from each other that guided modes do not ”see the other core regions”.

Core
Localized waveguide mode of the form
\[ E(r) = F(x, y)e^{i\beta z} \]

Continuum of cladding modes

Dielectric constant distribution for a supercell of size 8

The distance between neighbouring cores must be sufficient to avoid field overlap for the guided modes being calculated.
Notice how the field tends to avoid the air-holes

Localized waveguide mode of the form
\[ E(\mathbf{r}) = F(x, y)e^{i\beta z} \]

Dielectric constant distribution for a supercell of size 8

SiO₂/Water, D/\Lambda = 0.23
N_{supercell} = 16

Continuum of cladding modes

Waveguiding core region
Bandgap-guiding fibres: The honeycomb fiber

Honeycomb fiber

--- Supercell

Bandgap-guiding fibres: The honeycomb fiber

\[ \Gamma_s = 99\% \]

Bandgap-guiding fibres: The honeycomb fiber

Experimental pictures taken from J.C. Knight et al., Science, 282, 1476 (1998).
Same fiber with water in the air-holes
Bandgap-guiding fibres: The triangular fibre with large air-holes

Band diagrams of photonic crystal fiber-cladding material with water in the air-holes

\[ \frac{\beta \Lambda}{2\pi} = 3.393 \]
\[ f = 0.7 \]

\[ \frac{\beta \Lambda}{2\pi} = 4.63 \]
\[ f = 0.7 \]

\[ \frac{\beta \Lambda}{2\pi} = 5.604 \]
\[ f = 0.7 \]

\[ \frac{3.393}{2.55} = 1.33 \]
\[ \frac{4.63}{3.48} = 1.33 \]
\[ \frac{5.604}{4.21} = 1.33 \]
Simple cell (cladding)

5x5 supercell with large water core region

\[ \beta \Lambda/2\pi = 4.5 \Rightarrow \beta/k_0 \ (A/B) = 1.353 > 1.33 \Rightarrow \text{the mode is not primarily in the water} \]
\( \frac{\beta \Lambda}{2\pi} = 3.393 \Rightarrow \) there is a bandgap that can allow modes with a mode index equivalent to the refractive index of water, and such a mode is expected to be primarily confined in the water core region. Calculations show that there are also a large number of localized modes that are not primarily in the water core region similar to the previous slide (at least 10 even for this small core region).

Two degenerate core modes

Their sum

Supercell
Physical properties

- Core diameter 20 μm ± 2 μm
- Pitch (distance between cladding hole centers) 3.9 μm
- Air Filling Fraction in the holey region > 90%
- Diameter of holey region 73 μm
- Diameter of silica cladding 115 μm
- Coating diameter (single layer acrylate) 220 μm
- Available length up to 1 km