Lecture 9

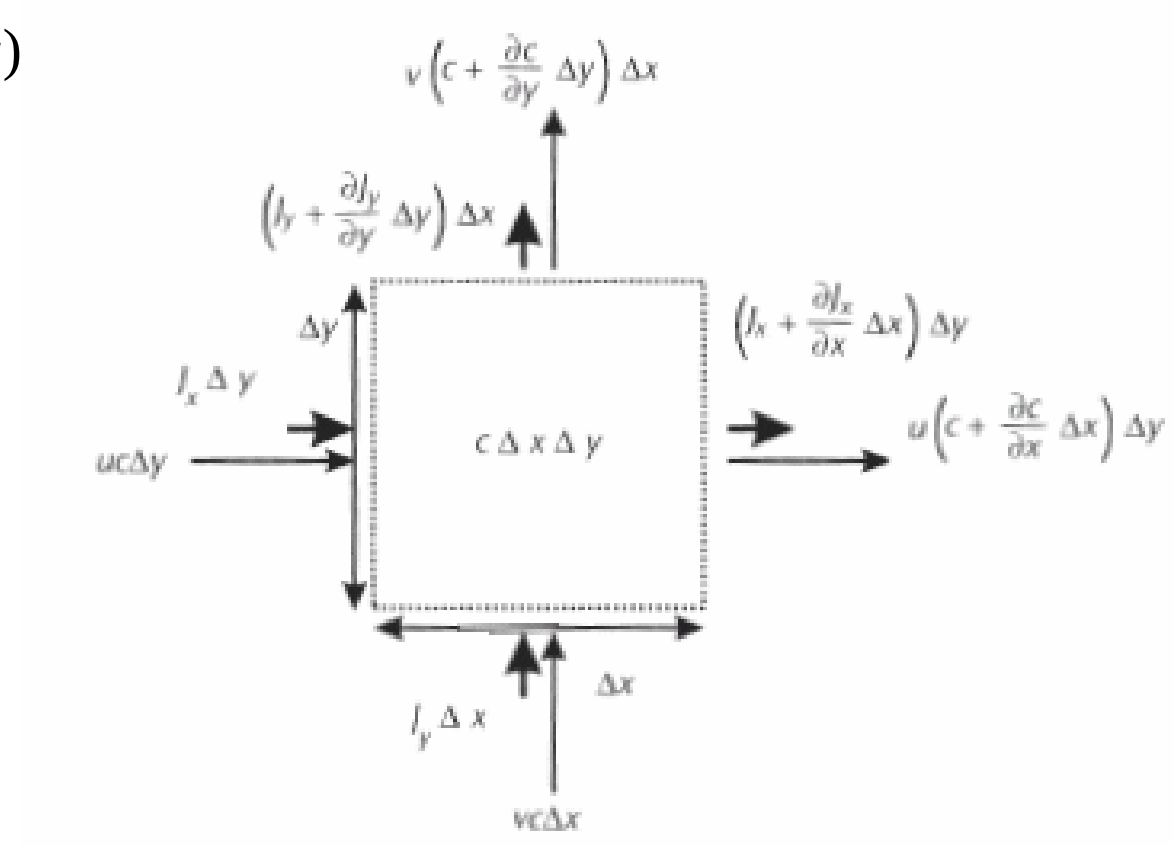
Diffusion in microchannels II.
Experimental Flow
Characterization Electrofluidics.

Advection-Diffusion Equation

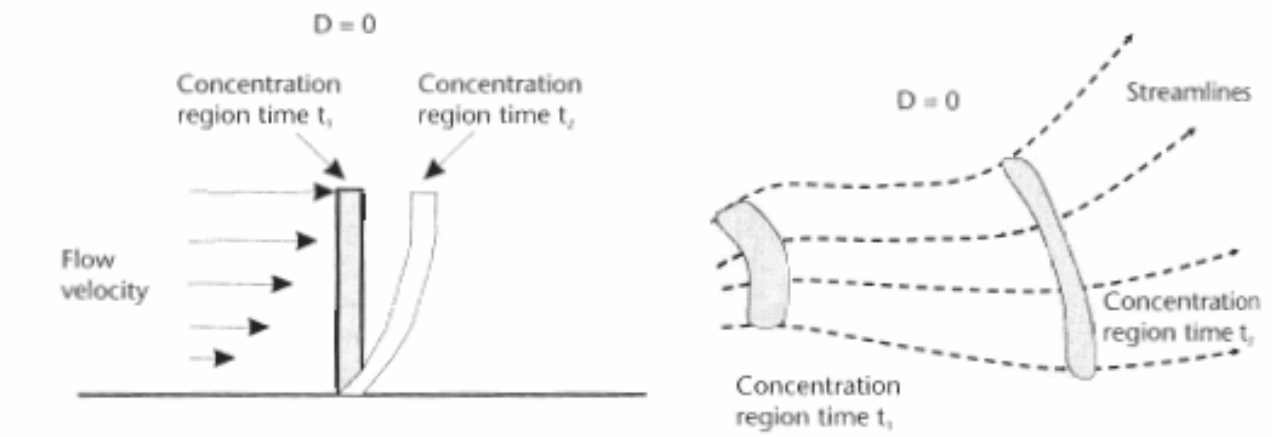
$$\frac{\partial c}{\partial t}\Delta x\Delta y + u\frac{\partial c}{\partial x}\Delta x\Delta y + v\frac{\partial c}{\partial y}\Delta x\Delta y + \frac{\partial J_x}{\partial x}\Delta x\Delta y + \frac{\partial J_y}{\partial y}\Delta x\Delta y = 0$$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = \nabla\cdot(D\nabla c)$$

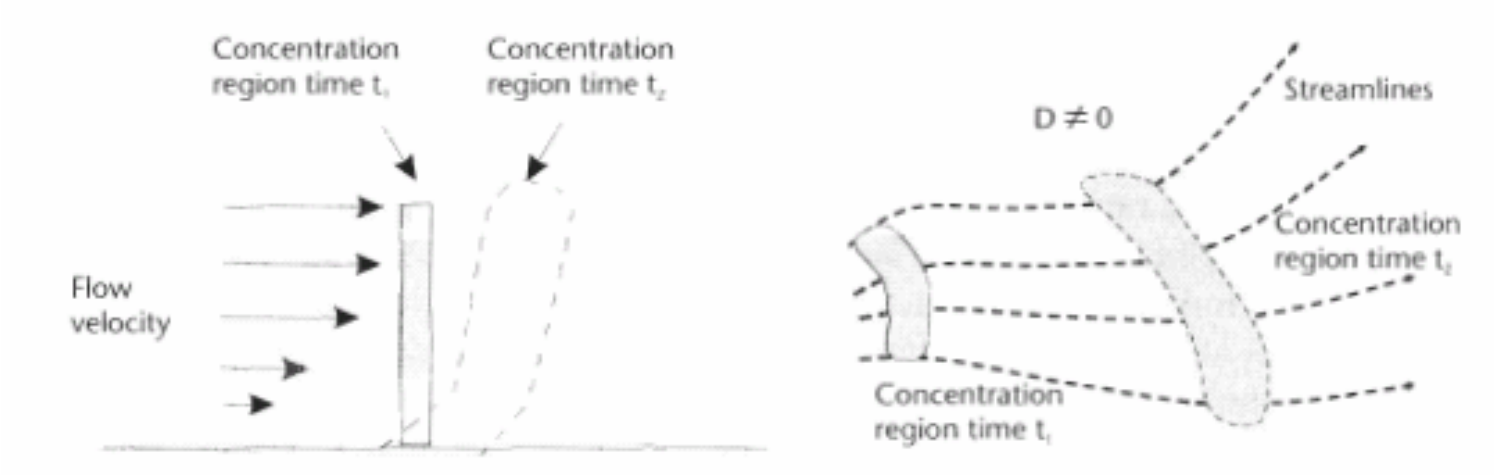
$$\frac{Dc}{Dt} = \nabla\cdot(D\nabla c)$$



Advection-Diffusion equation



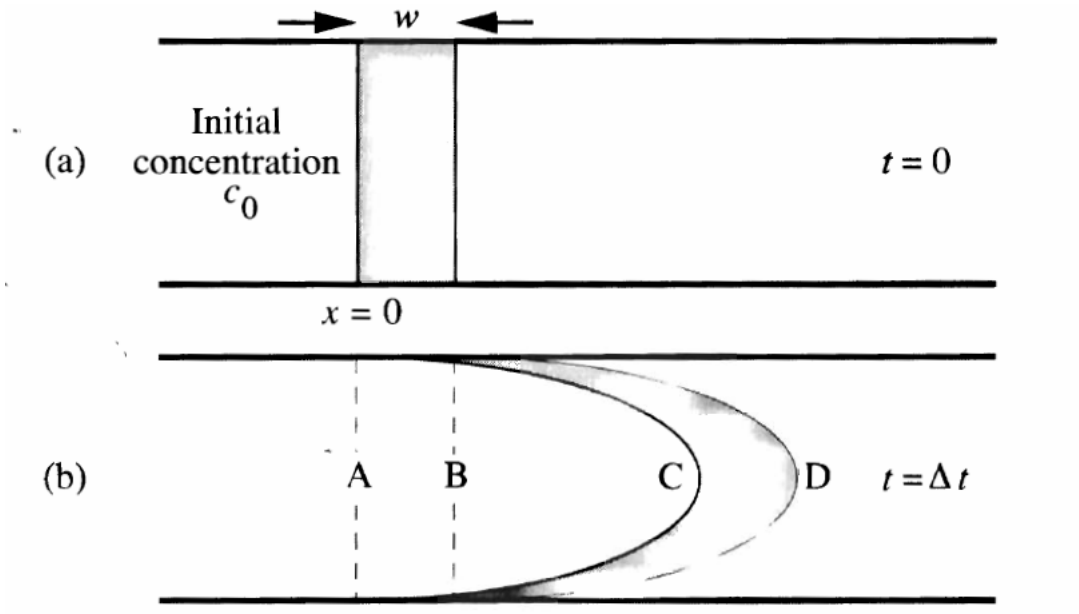
- D=0: concentration remains the same along a trajectory



- D≠0: concentration spreads with time

Taylor dispersion in pressure-driven flow

- material transported in a pressure-driven laminar flow will be spread due to **diffusion** and **flow profile**



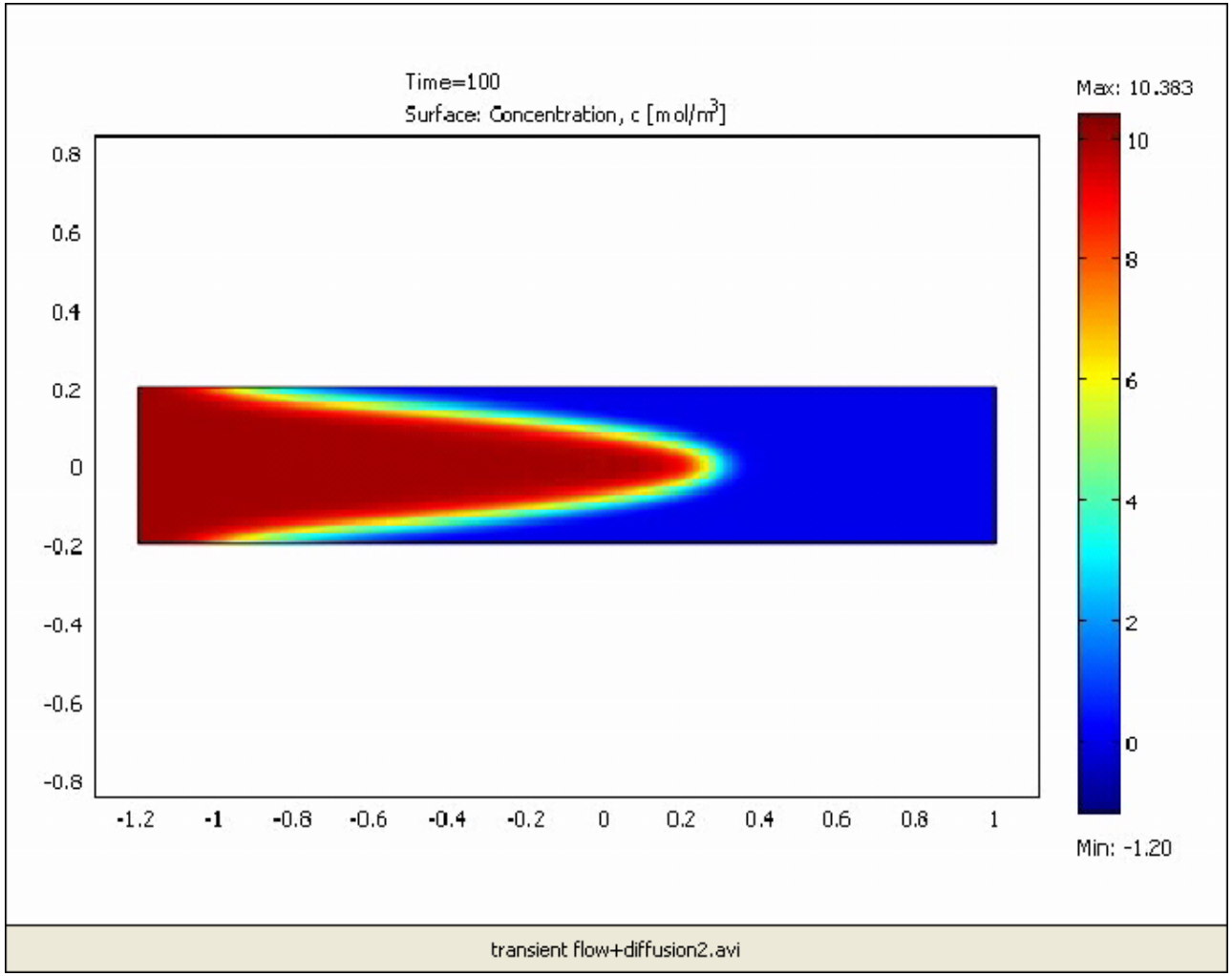
Observation: sample band width changes is proportional to the square root of time, but with “effective” diffusion coefficient:

$$\frac{\partial c}{\partial t} = D\left(1 + \frac{Pe^2}{48}\right)\frac{\partial^2 c}{\partial x^2}$$

where Peclet number $Pe = \frac{U_0 a}{D}$

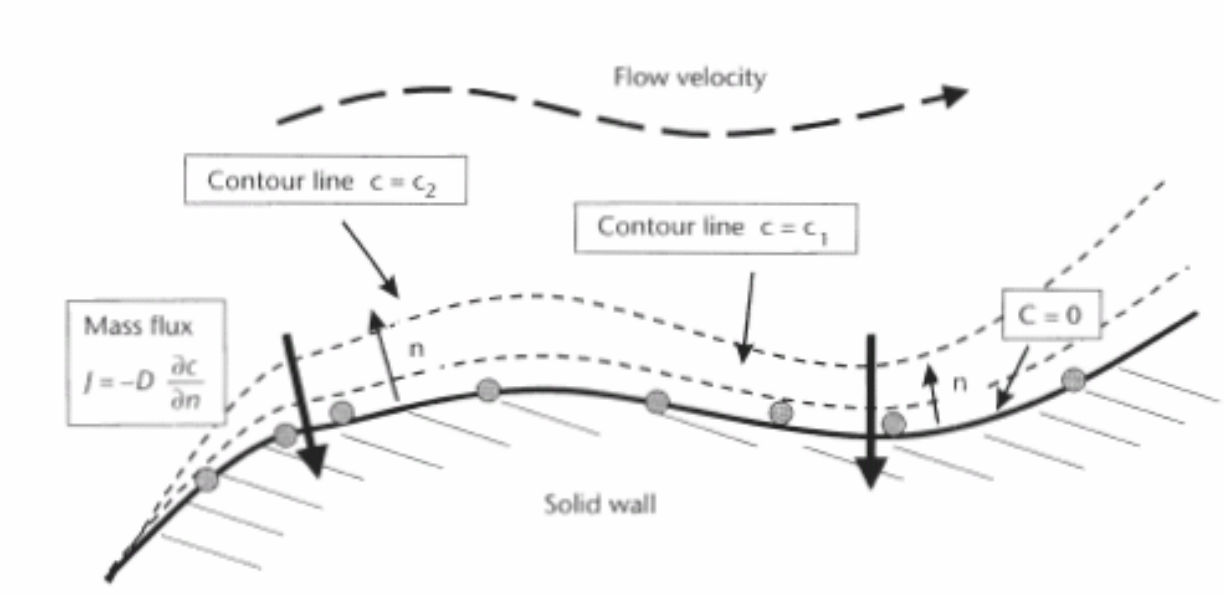
Modeling of time-dependent flow

- Model: spreading of a step distribution
- Parameters: D=1e-5; u=0.01 m/s



Advection-Diffusion equation

- Boundary conditions:
  - Dirichlet condition: c=0 at the wall



  - homogeneous Neumann condition: $\partial c/\partial n=0$ at the wall – absence of mass flux to the wall.
  - Neumann condition:

$$J_n = -D\frac{\partial c}{\partial n} = \frac{d\Gamma}{dt}$$

reaction kinetics

Coupling with Hydrodynamics

- The advection-diffusion equation requires knowledge of the velocity field; the complete set of equations:

$$\frac{\partial \rho}{\partial t} + \nabla\left(\rho \vec{U}\right) = 0$$

$$\rho \frac{\partial \vec{U}}{\partial t} + \rho \vec{U} \cdot \nabla \vec{U} = -\nabla P + \eta \Delta \vec{U} + F$$

$$\frac{\partial c}{\partial t} + \vec{U} \cdot \nabla c = \nabla \cdot \left(D \nabla c\right) + S$$

the system contains: 5 unknowns (u, v, w, P, c), 2 fluid properties and 2 external actions (F and S).

Peclet number

- diffusion advection equation without source or sink:

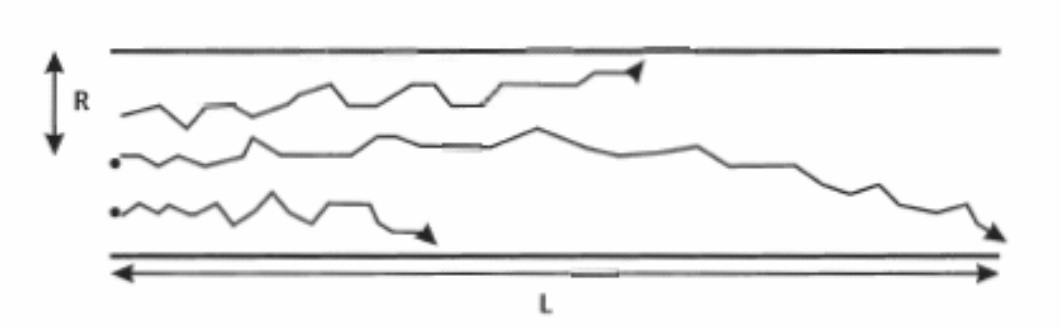
$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}=D\left[\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right]$$

- the equation has 4 parameters: U_∞, L,c_0 and D. From Pi theorem, there is one nondimensional number:

$$Pe=\frac{U_\infty L}{D}$$

Distance of capture in a capillary

- by comparing axial convection to radial diffusion

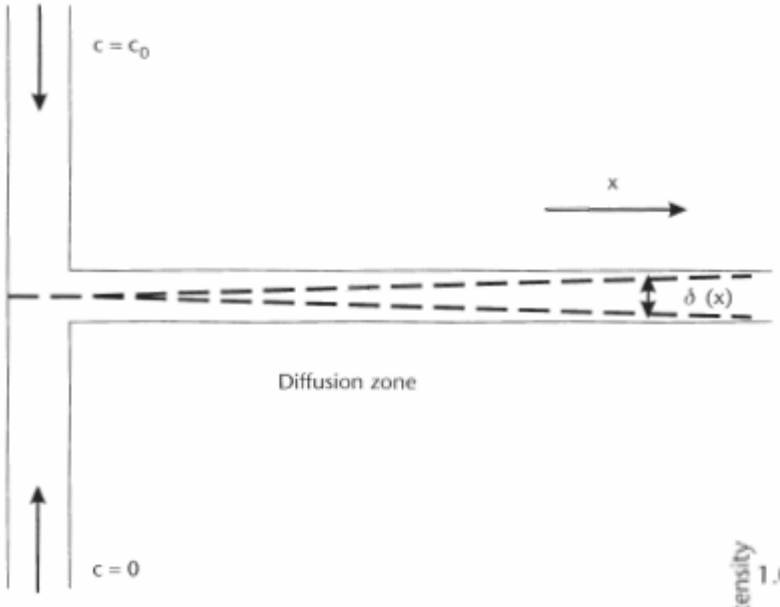


$$\tau \approx \frac{R^2}{4D} \qquad L \approx 2V\tau \approx \frac{VR^2}{2D} \qquad L \approx \frac{Q}{2\pi D}$$

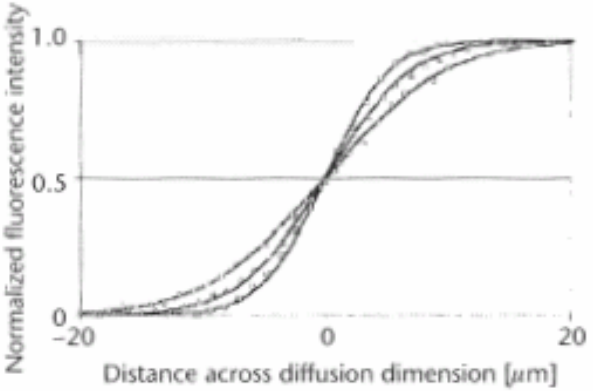
$$\frac{L}{R} = \frac{1}{2}Pe$$

Determination of diffusion coefficient

- Microfluidic systems are convenient to determine diffusion coefficients in liquid phase



$$c(x,y) = \frac{1}{2} c_0 \left(1 - erf \frac{y\sqrt{U}}{\sqrt{4Dx}} \right)$$



Mixing of fluids

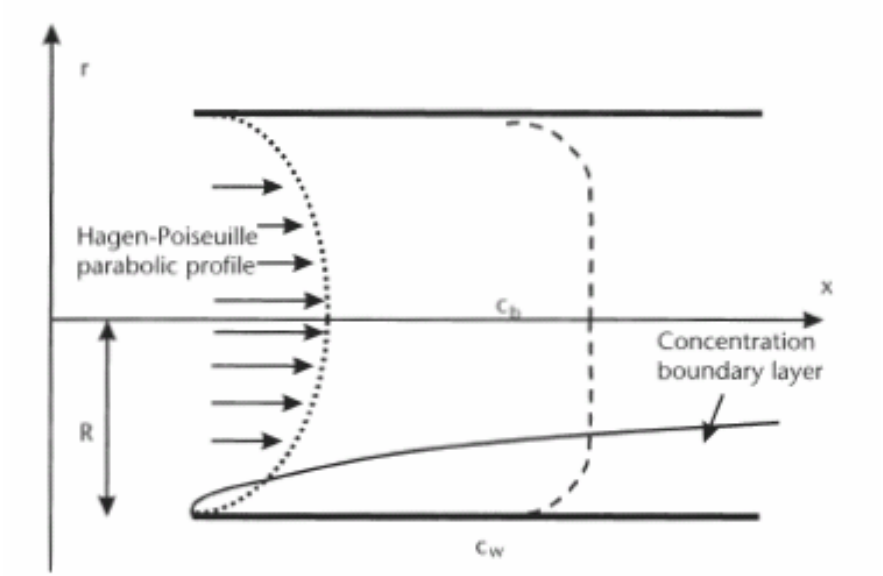
- High laminarity of microflow delays mixing of the components
- to benefit advantages of microfluidics (e.g. reduction of reaction rates) mixing should be accelerated

- let's estimate a distance L at which a relative concentration on the wall is 90% of $c_0/2$.

$$c(x,y) = \frac{1}{2} c_0 \left(1 - erf \frac{y\sqrt{U}}{\sqrt{4Dx}} \right) \qquad \frac{R\sqrt{U}}{\sqrt{4DL}} = erfinv(0.1)$$

$$\frac{L}{R} \approx 32 \frac{UR}{D} = 32 Pe$$

Concentration boundary layer



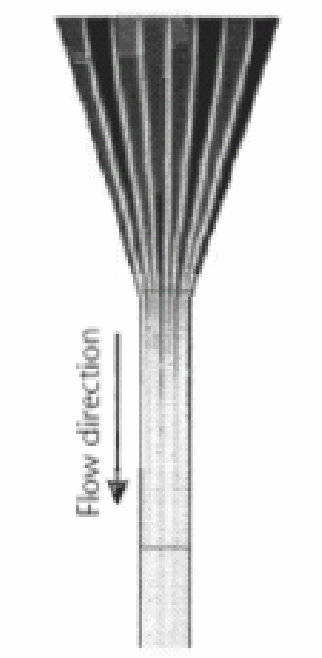
$$L_{mix} \approx 32R \cdot Pe$$

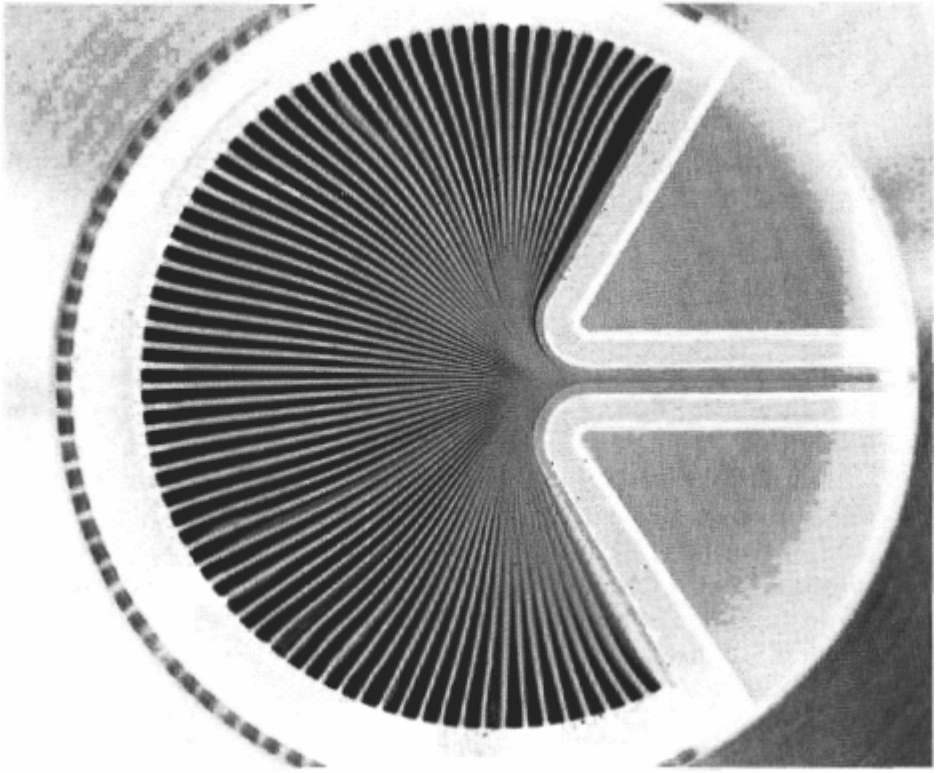
$$h \approx 0.04 \cdot 2R \cdot \mathrm{Re}_D$$

- hydrodynamic entrance: the action of viscosity homogenizes the flow to reach a fully developed flow. Very short in microfluidics.
- mixing: the action of molecular diffusion homogenizes concentration

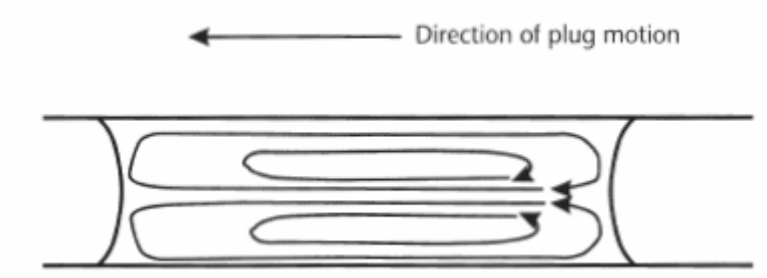
Improving the mixing

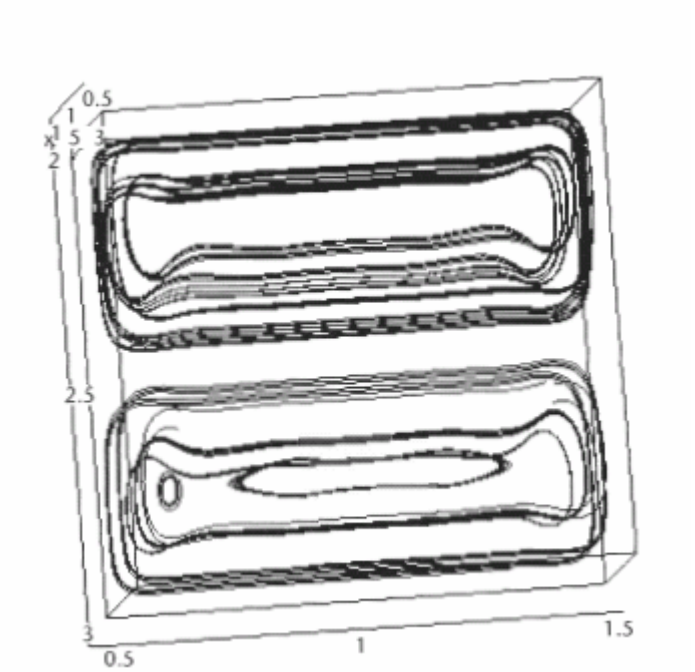
- as mixing length varies as square of R, dividing flow into thin brunches (achieved by lamination and hydrodynamic focusing) improves mixing





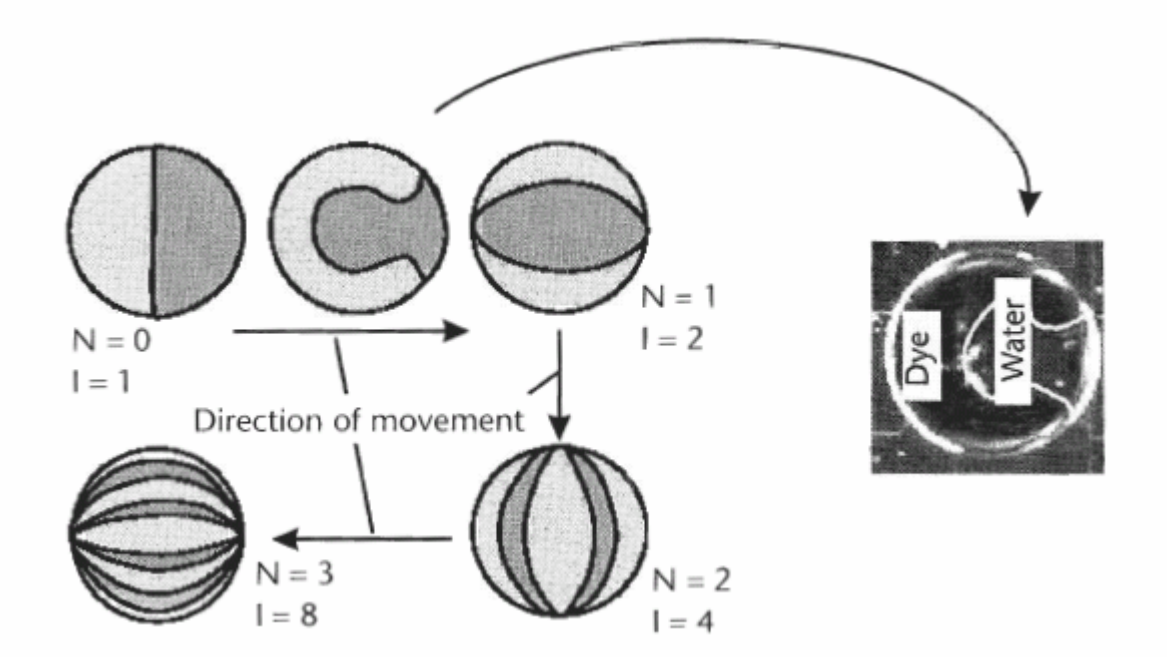
Mixing in two phase flow



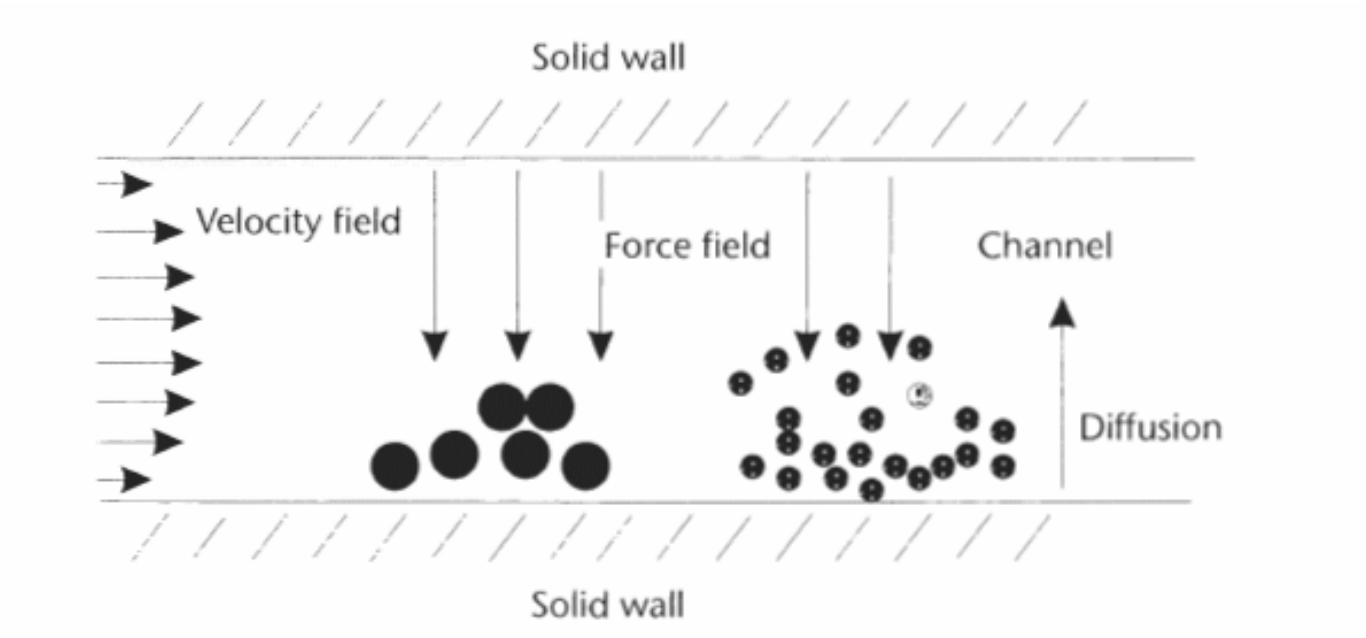


Mixing in digital microfluidics

- mixing in digital microfluidics shows very special pattern

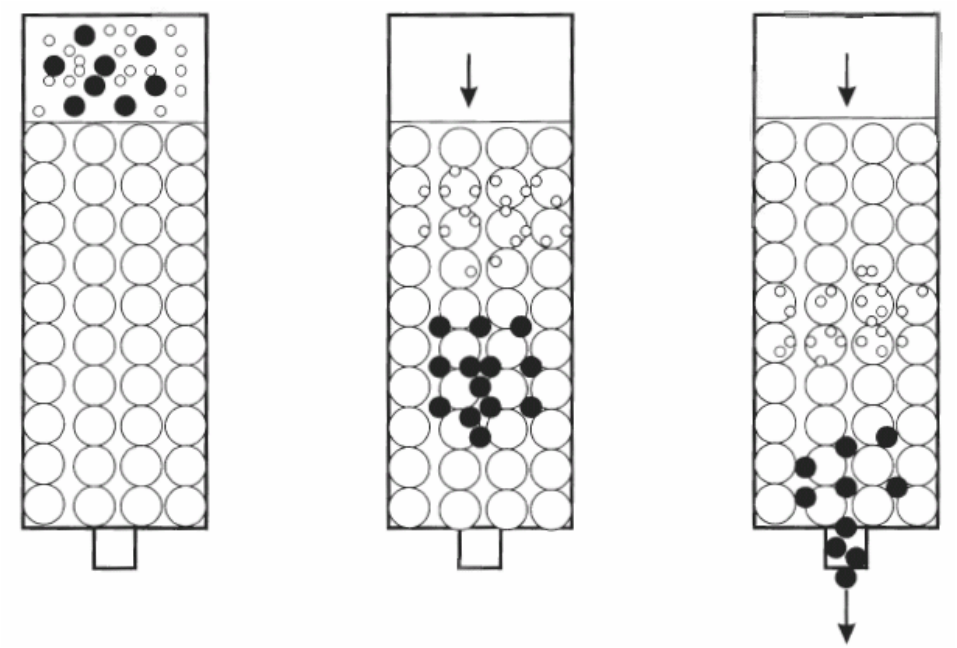


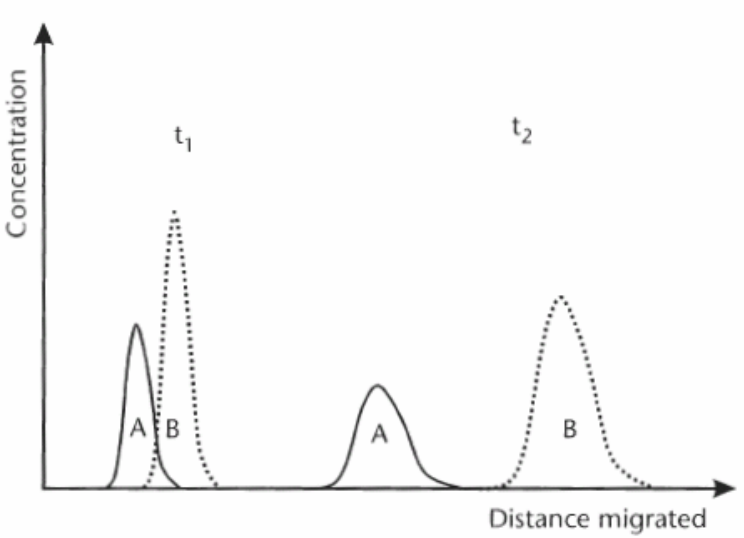
Separation/purification of bioparticles



- Field Flow Fractionation (FFF): particles in a liquid flow separate according to their physical properties.

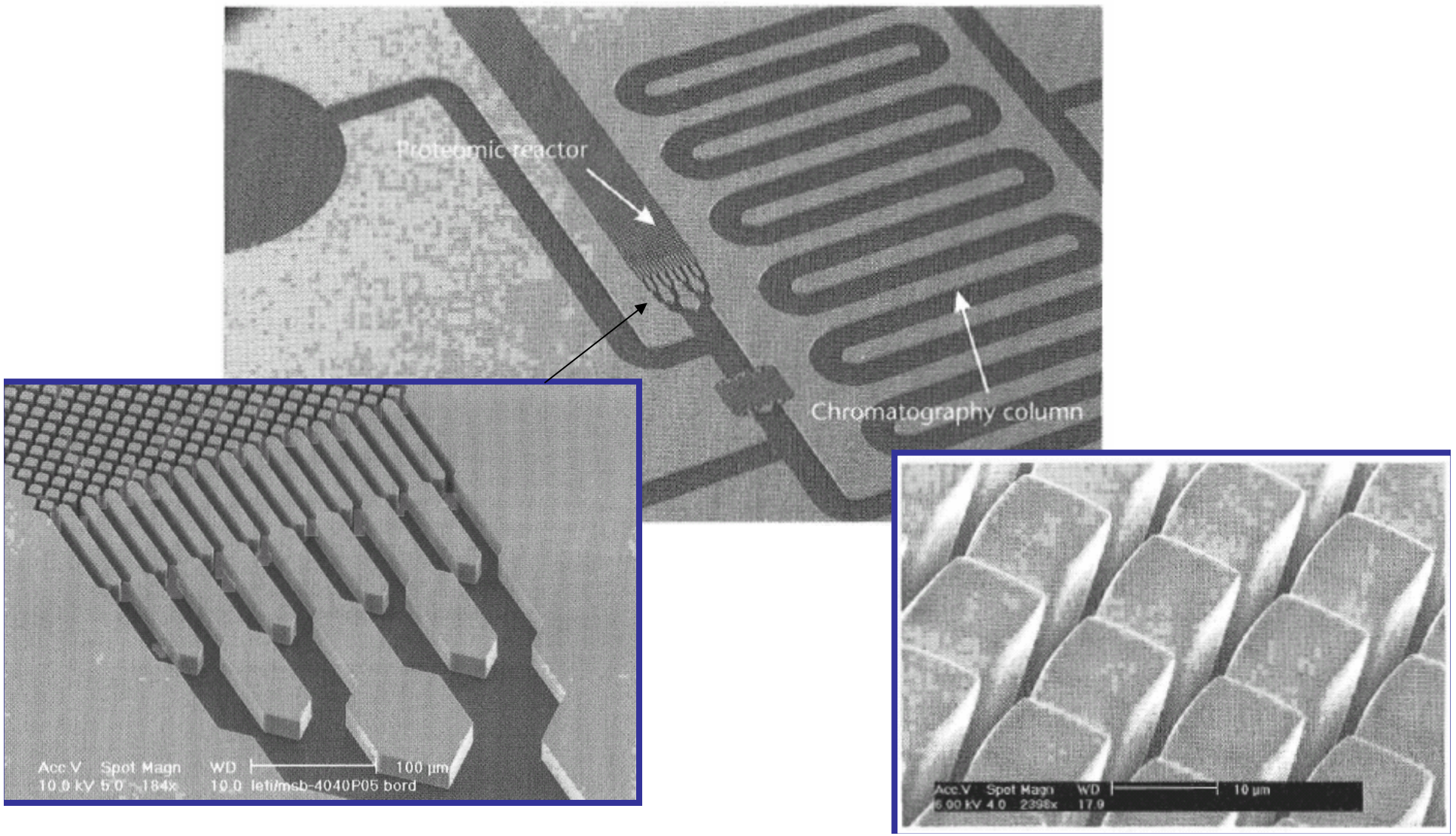
Chromatography



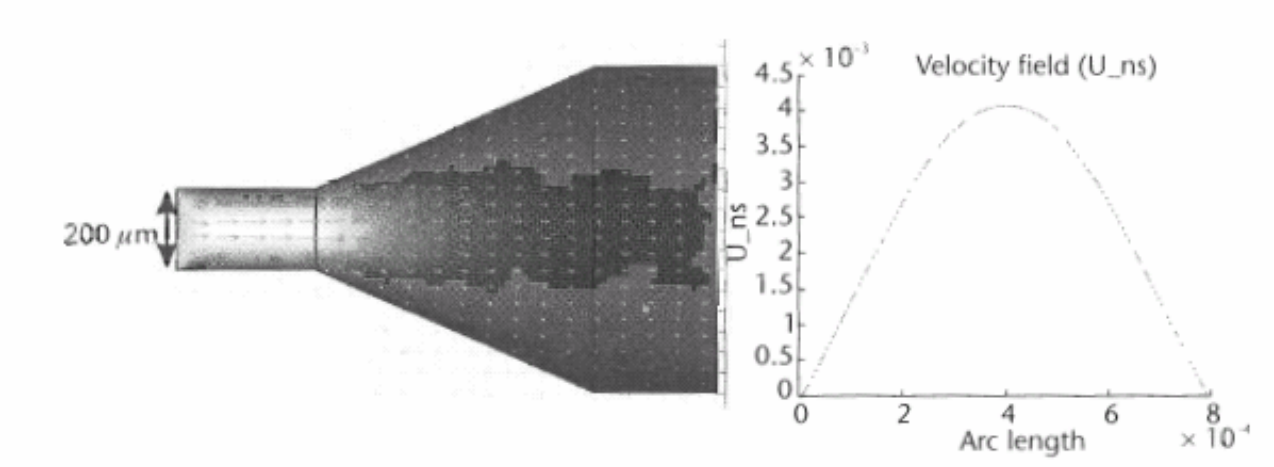


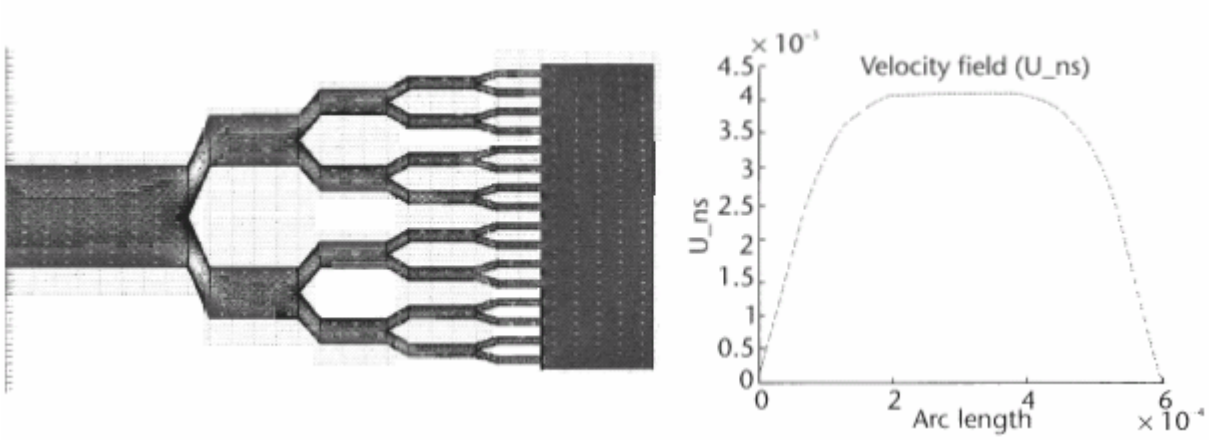
- Chromatography devices include two phases: mobile (transporting the sample) and stationary. Stationary phase is designed to retain longer the phase it associates with (e.g small fractions in size exclusion chromatography)

Proteomic reactor



Proteomic reactor





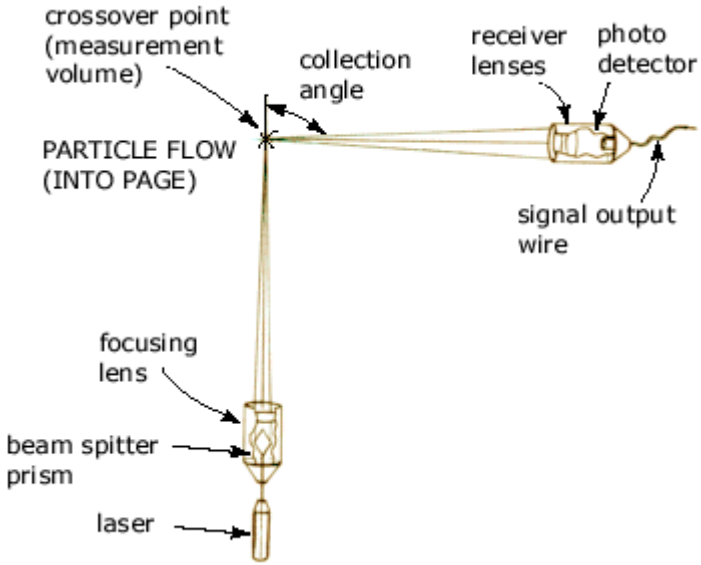
Microfluidics: Experimental Flow Characterization

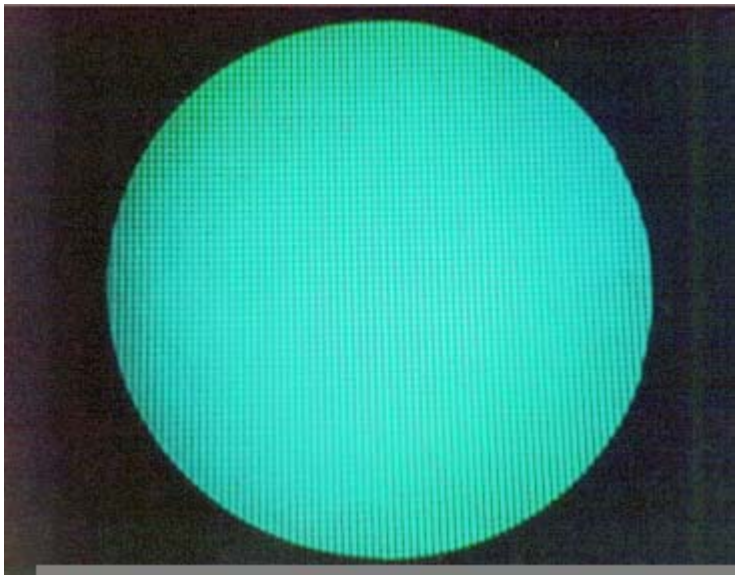
Why knowing flow field is important

- Microfluidics devices used in variety if applications, e.g. Inkjet nozzles, medical microfluidic devices for diagnostics, PCR based DNA amplification/detection, channels for DNA fractinations, microtrusters on satellites and microaircrafts
- complicated geometries, flow difficult to simulate numerically

Flow characterization techniques

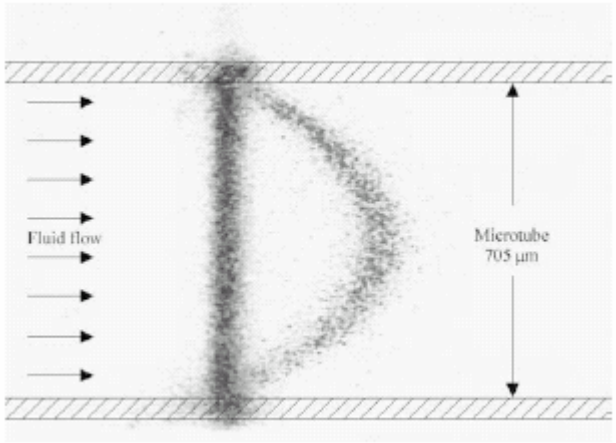
- Pointwise methods
  - Laser Doppler velocimetry (LDV). Lateral x longitudinal resolution down to 5um x 10 um
  - Optical Doppler tomography: suitable for highly scattered medium (e.g. Blood flow under the skin), resolution down to 5um x 15 um

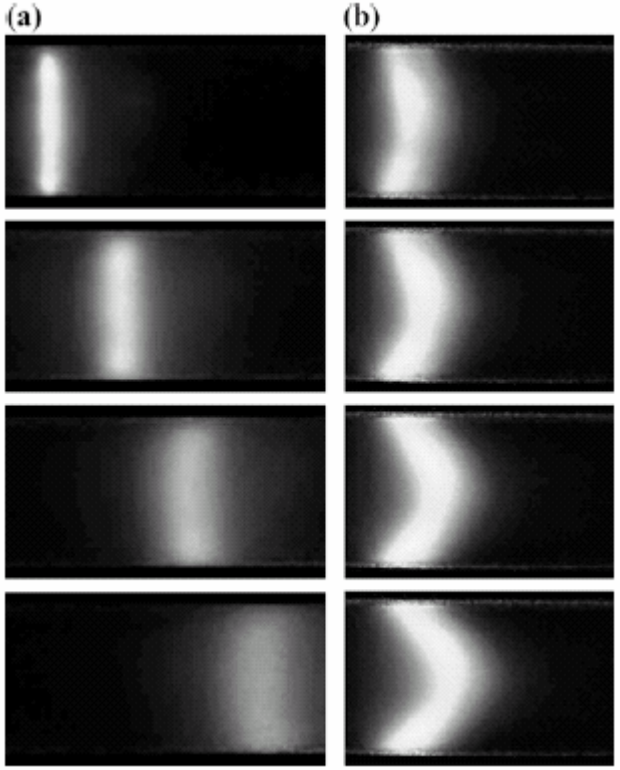




Flow characterization techniques

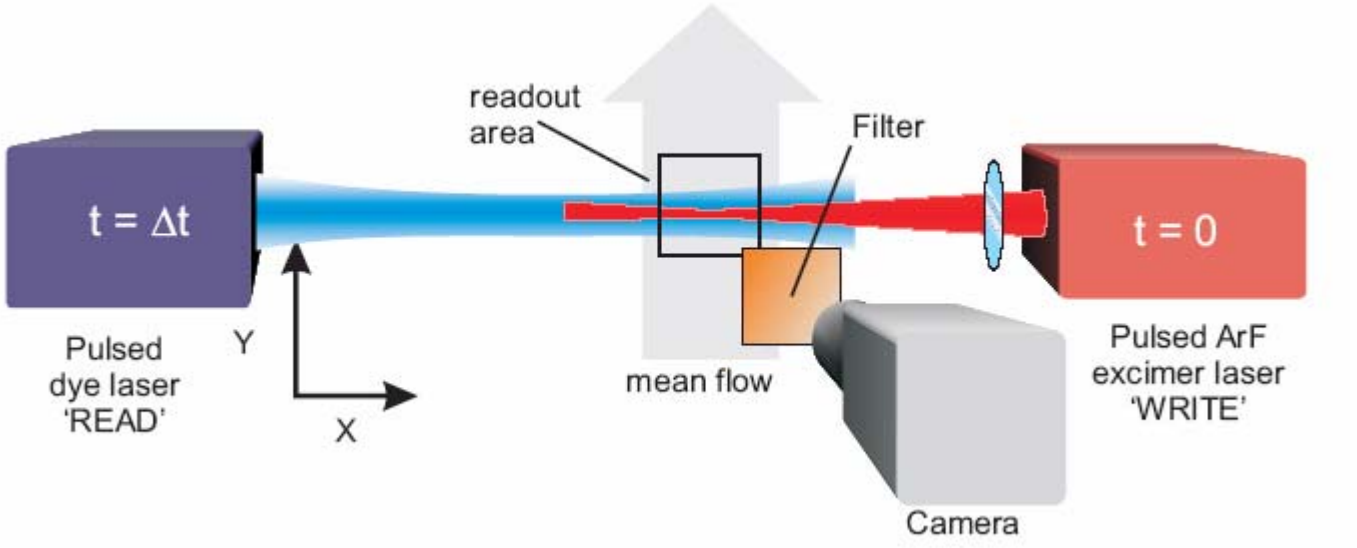
- Full-Field techniques: generate two component velocities distributed in 2D plane. Essential for microfluidics
  - Scalar Image Velocimetry (SIV): uses molecular tracers to visualize the flow. Can go through smallest channels, insensitive to electric forces, but subject to intensive diffusion. Variation: dye uncaging

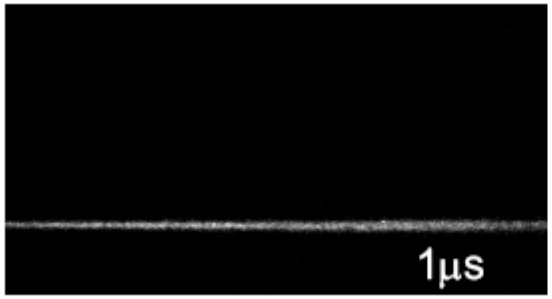


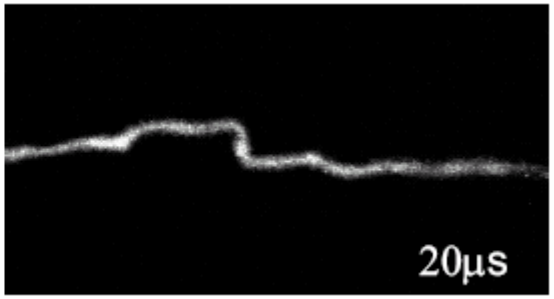


Flow characterization techniques

– Molecular tagging velocimetry: fluorescent/phosphorescent dye excited in a patteren, evolution of the pattern followed







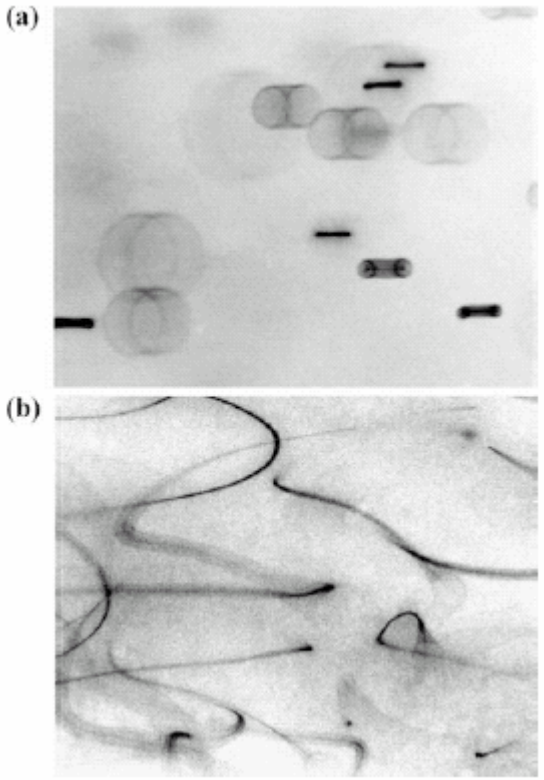
from: Thijs Elenbaas, PhD Thesis, Univ. Eindhoven '06

Flow characterization techniques

- Particle Streak Velocimetry
- Particle Image Velocimetry (PIV)

Advantages:

- high spacial resolution
- large range of velocities covered (up to 8m/s)
- simplicity



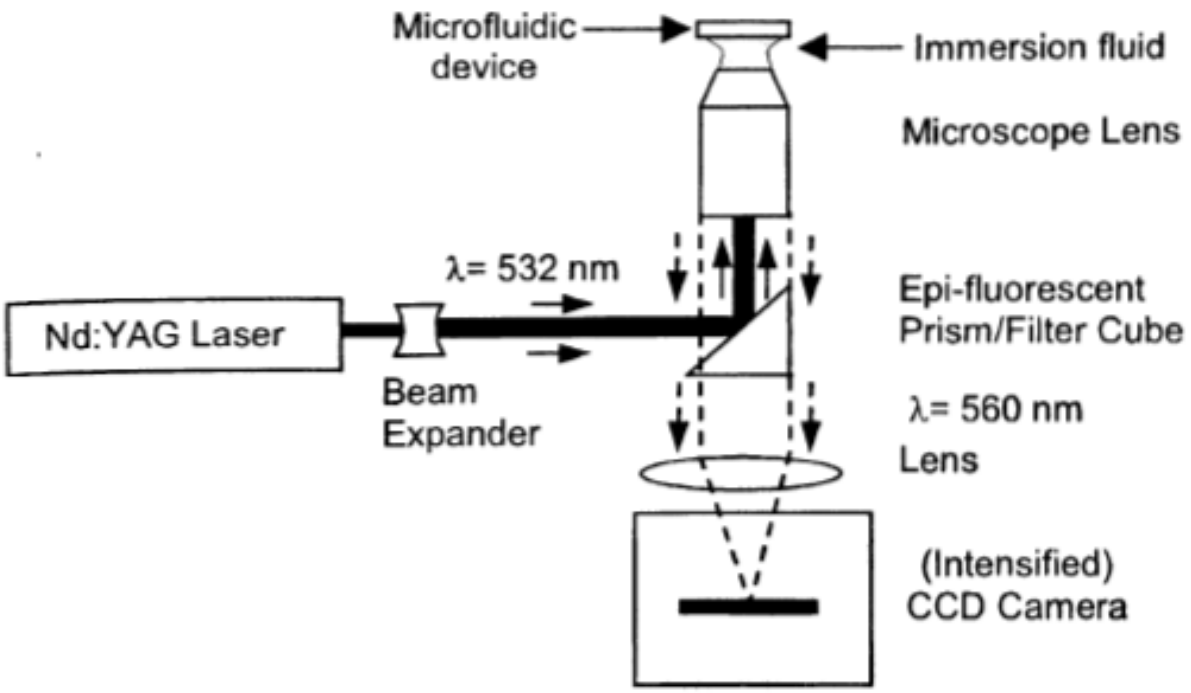
Comparison of High-Resolution Velocimetry Techniques [22].

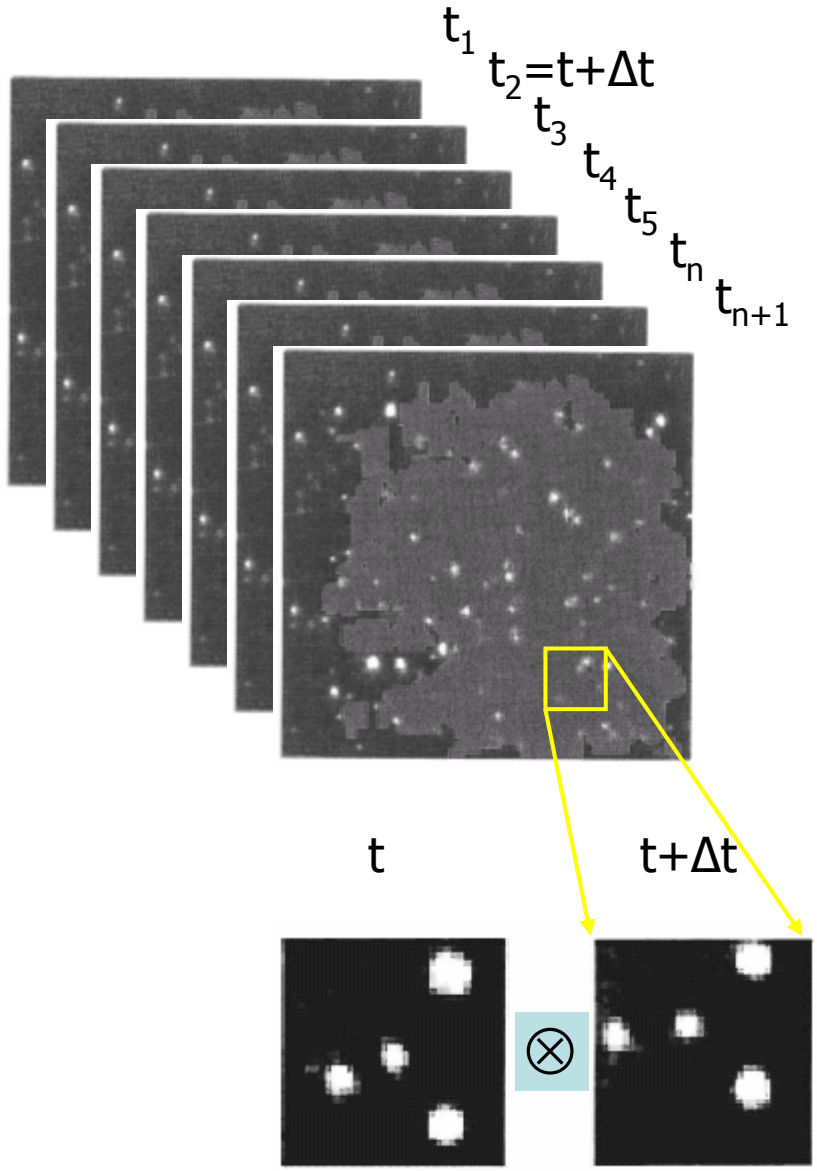
Technique	*Author*	*Flow Tracer*	*Spatial Resolution (μm)*	*Observation*
LDA	Tieu et al. (1995)	_____	5 × 5 × 10	4–8 fringes limits velocity resolution
Optical Doppler tomography (ODT)	Chen et al. (1997)	1.7-μm polystyrene beads	5 × 15	Can image through highly scattering media
Optical flow using video microscopy	Hitt et al. (1996)	5-μm blood cells	20 × 20 × 20	In vivo study of blood flow
Optical flow using X-ray imaging	Lanzillotto et al. (1996)	1 – 20-μm emulsion droplets	~ 20 - 40	Can image without optical access
Uncaged fluorescent dyes	Paul et al. (1997)	Molecular Dye	100 × 20 × 20	Resolution limited by molecular diffusion
Particle streak velocimetry	Brody et al. (1996)	0.9-μm polystyrene beads	~ 10	Particle streak velocimetry
PIV	Urushihara et al. (1993)	1-μm oil droplets	280 × 280 × 200	Turbulent flows
Super-resolution PIV	Keane et al. (1995)	1-μm oil droplets	50 × 50 × 200	Particle tracking velocimetry
Micro-PIV	Santiago et al. (1998)	300-nm polystyrene particles	6.9 × 6.9 × 1.5	Hele-Shaw Flow
Micro-PIV	Santiago et al. (1998)	300-nm polystyrene particles	6.9 × 6.9 × 1.5	Silicon microchannel flow
Micro-PIV	Meinhart et al. (1999)	200-nm polystyrene particles	5.0 × 1.3 × 2.8	Microchannel flow

Micro Particle Image Velocimetry

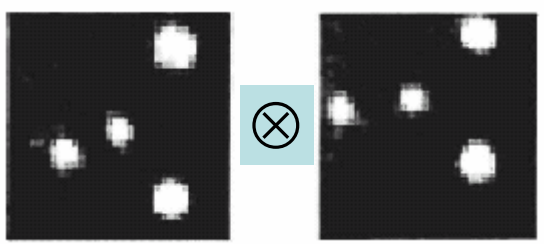
- Particle Image Velocimetry (PIV):
  - flow is seeded by small particles,
  - consecutive photographs of particles distribution are made
  - Images are sectioned into ***interrogation regions***
  - Motion of particles within interrogation region determined by image cross-correlation

Micro Particle Image Velocimetry

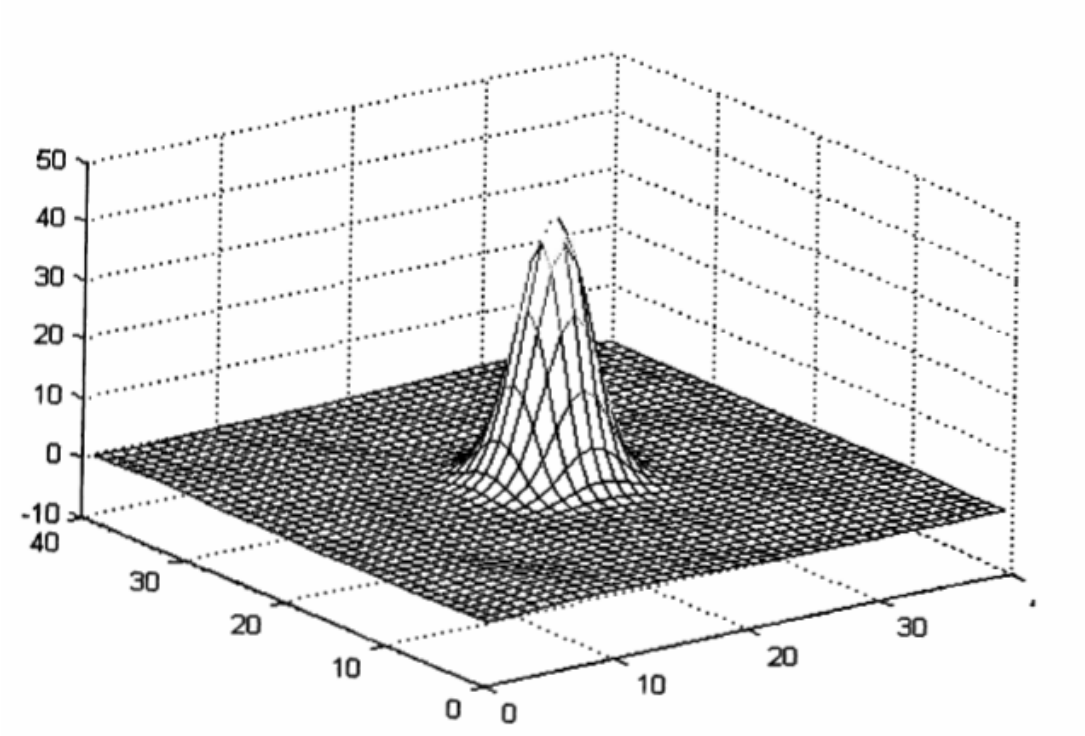



t_1
$t_2=t+\Delta t$
t_3
t_4
t_5
t_n
t_{n+1}
t
t+Δt
⊗

t t+Δt



$$\Phi(m,n) = \sum_{j=1}^{q} \sum_{i=1}^{p} f(i,j) \cdot g(i+m, j+n)$$



Physics consideration of micro-PIV

- Particles small considered to λ. If d<<λ, amount of light scattered varies as d^6.
- Brownian motion

Flow particle dynamics

- Response time of the paricle

$$\tau_p = \frac{d_p{}^2 \rho_p}{18\eta}$$

  - E.g. for 300nm polystyrene in water, approx 10^{-9} s

- Example: calculate responce time for polystyrene particles (ρ=1.05 g/cm^3), diameters 1um, 300nm, 100nm in water (μ=10^{-3} kg/m s)

Velocity errors

- Particle is subject to Brownian motion
- High frequency velocity components of Brownian motion are not resolved, velocity meant as average displacement (after many fluctuations) per given time

$$\langle x \rangle^2 \propto D\Delta t$$

For spherical particle:

$$D = \frac{KT}{3\pi\mu d_p}$$

Probablilty of particle displacement is described by 3D Gaussian

$$p(x,y,z) = \frac{\exp[-(x^2+y^2+z^2)/4D\Delta t}{(4\pi D\Delta t)^{3/2}}$$

PIV measurements are done in 2D within some measurement depth:

$$p(x,y) = \frac{\exp[-(x^2+y^2)/4D\Delta t}{4\pi D\Delta t}$$

That's an approximation as the diffusion is still 3D and a particle can leave the measurement volume in z direction. Valid if $w\Delta t < \delta z_m$.

Brownian motion causes particle trajectories to fluctuate about the ideal pathlines.
Imagine ideal streamline followed for a period Δt:

$$\Delta x = u\Delta t$$

$$\Delta y = v\Delta t$$

Relative errors due to Brownian motion:

$$\varepsilon_x = \frac{\sigma_x}{\Delta x} = \frac{1}{u}\sqrt{\frac{2D}{\Delta t}}$$

$$\varepsilon_y = \frac{\sigma_y}{\Delta y} = \frac{1}{v}\sqrt{\frac{2D}{\Delta t}}$$

More significant for short measurement time slow velocities.

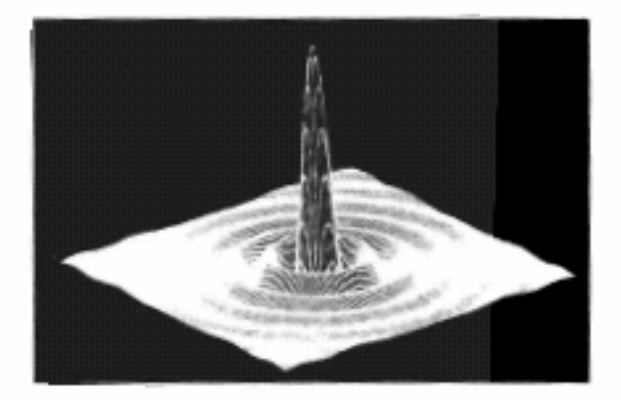
Velocity errors

Positional uncertainty due to Brownian motion:
particle displacement during the exposure time

Example: Compute dffusion coefficient D for 300nm polystyrene particle in water at room temperature. Given a flow speed of 10mm/s and 10 particles in an interroagation region, what Δt is required to achieve 10% relative error? 1% relative error? What particle smearing would you expect at 10ms exposure time?

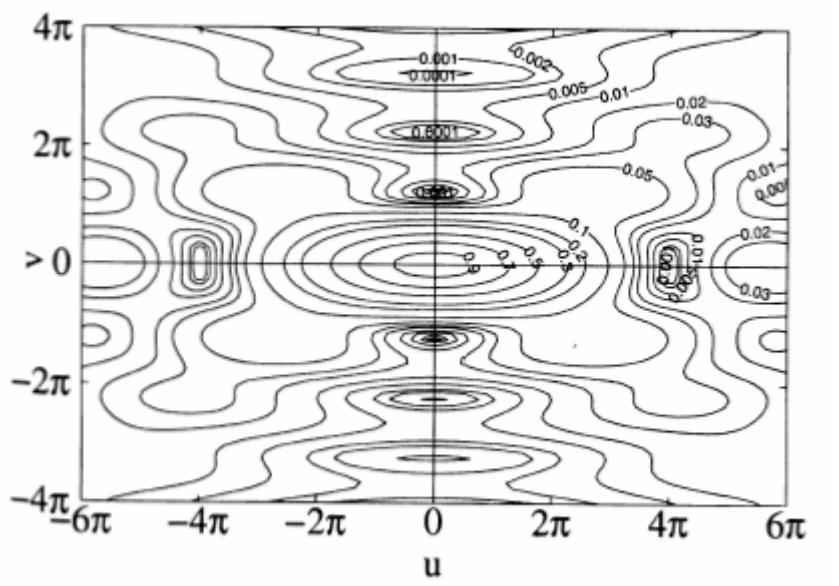
- Density of polystyrene $\rho=1.05$ g/cm^3 ,
- Viscosity of water $\mu=10^{-3}$ kg/m s

3D diffraction pattern



2D Airy pattern

$$I(u,0) = \left[\frac{\sin u/4}{u/4}\right]^2 I_0$$



$$I(0,v) = \left[\frac{2J_1(v)}{v}\right]^2 I_0$$

Depth of field

- Defined as ¼ of the distance between two minima, u=+-π

$$\delta z = \frac{n\lambda_0}{NA^2} + \frac{ne}{NA \cdot M}$$

Pixel spacing

Total magnification

Depth of correlation

- The distance away from focal plane where intensity of the particle considered to be sufficiently small (usually 0.1 of in focal intensity)

$$\delta z_m = \frac{3n\lambda_0}{NA^2} + \frac{2.16d_p}{\tan(\theta)} + d_p$$

Example: what depth of correlation would you expect for x40 lens, 0.7 NA, illumination at 600nm for 1um, 300nm and 100nm particles.

Background noise

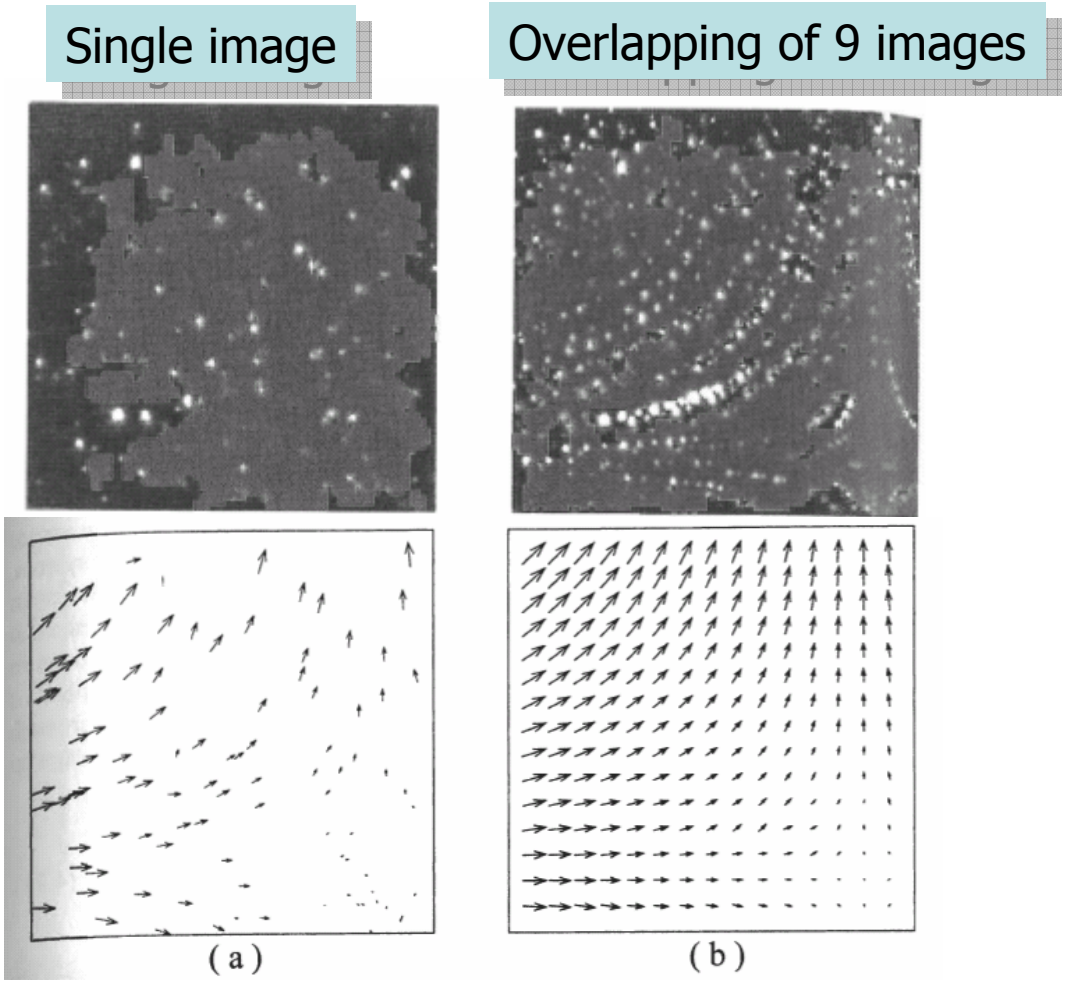
- Mainly due to particles out of focus.
  Can be lowered by reducing concentration of particles and depth of filed

The Effect of Background Noise on Image Quality (i.e., Signal-to-Noise Ratio) [23]

Depth (μm)	*Particle Concentration (by Volume)*			
	0.01%	*0.02%*	*0.04%*	*0.08%*
25	2.2	2.1	2.0	1.9
50	1.9	1.7	1.4	1.2
125	1.5	1.4	1.2	1.1
170	1.3	1.2	1.1	1.0

Processing methods for micro-PIV

- Overlapping of low image density (LID) PIV: single image pair might exhibit noise, it will be averaged out if more images are considered



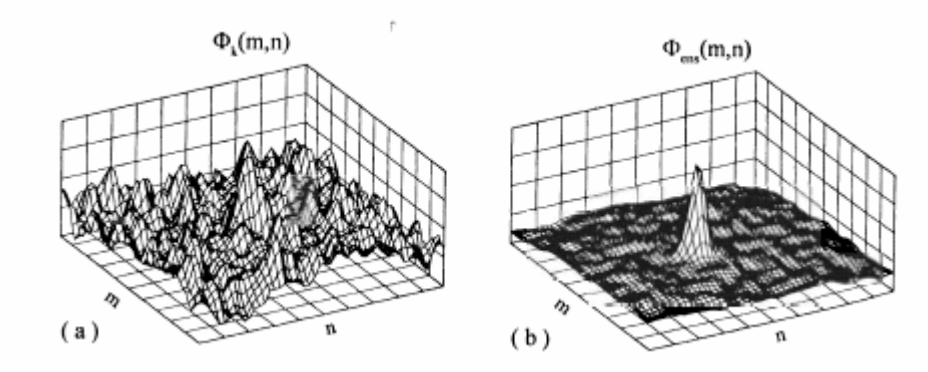
Correlation based overlapping

$$\Phi(m,n) = \sum_{j=1}^{q}\sum_{i=1}^{p} f(i,j)\cdot g(i+m, j+n)$$

In steady laminar flow, the peak position in correlation should be the same. Therefore:

$$\Phi_{ens}(m,n) = \frac{1}{N}\sum_{k=1}^{N}\Phi_k(m,n)$$

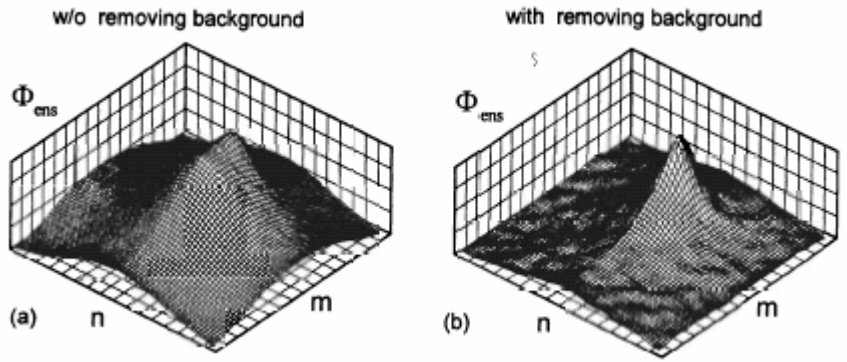
Single image pair



Averaged on 101 image pair

- Background removal
  - Averaging background over many images

    or
  - Using minimum value for every pixel in sequence



Micro-PIV examples

- Measurements in microchannel

Equipment:

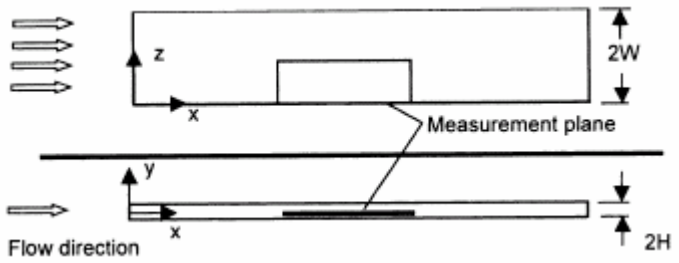
• 30um x 300um x 25mm glass channel

•Epifluorescent microscope with x60 lens, NA=1.4. Focal plane 7.5um above the bottom of the channel

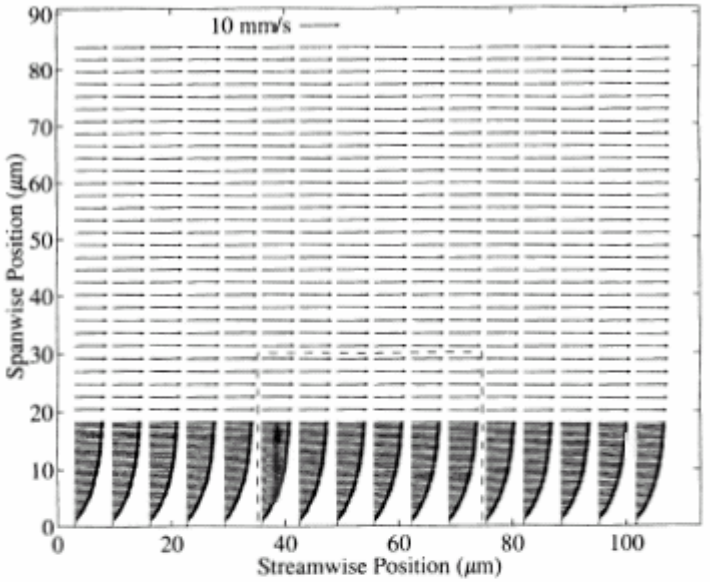
• light source: pulsed NdYAG laser

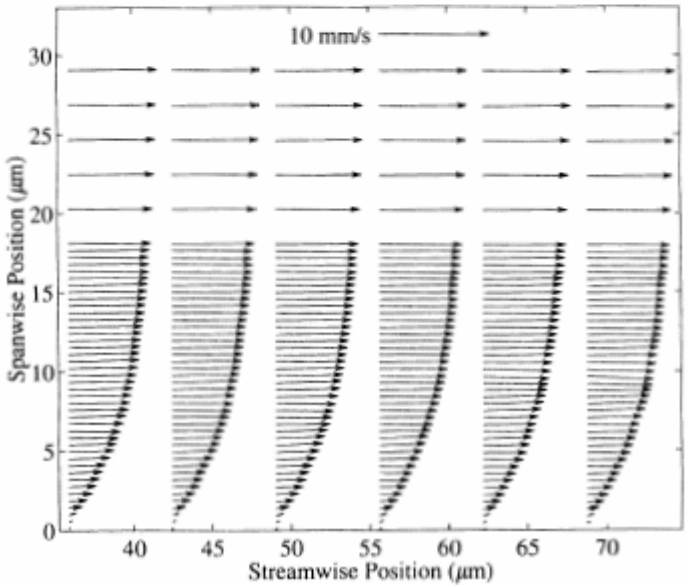
• Syringe pump. Flow rate 200ul/h

• 200nm fluorescent polystyrene particles



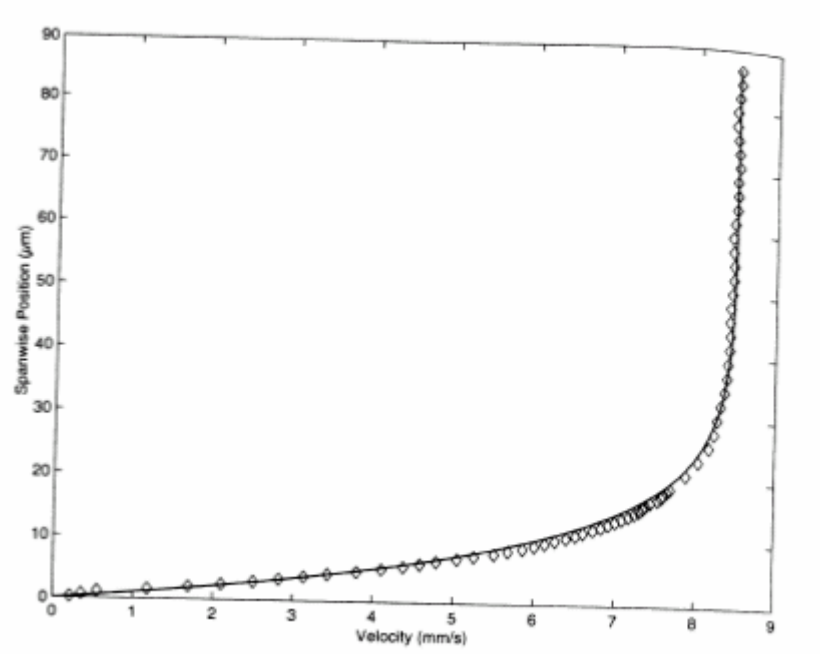
Interrogation window size:
120 x 8 pixels near the wall (13.6um x 0.9 um),
120 x 40 pixels everywhere else (13.6 x 40 um)



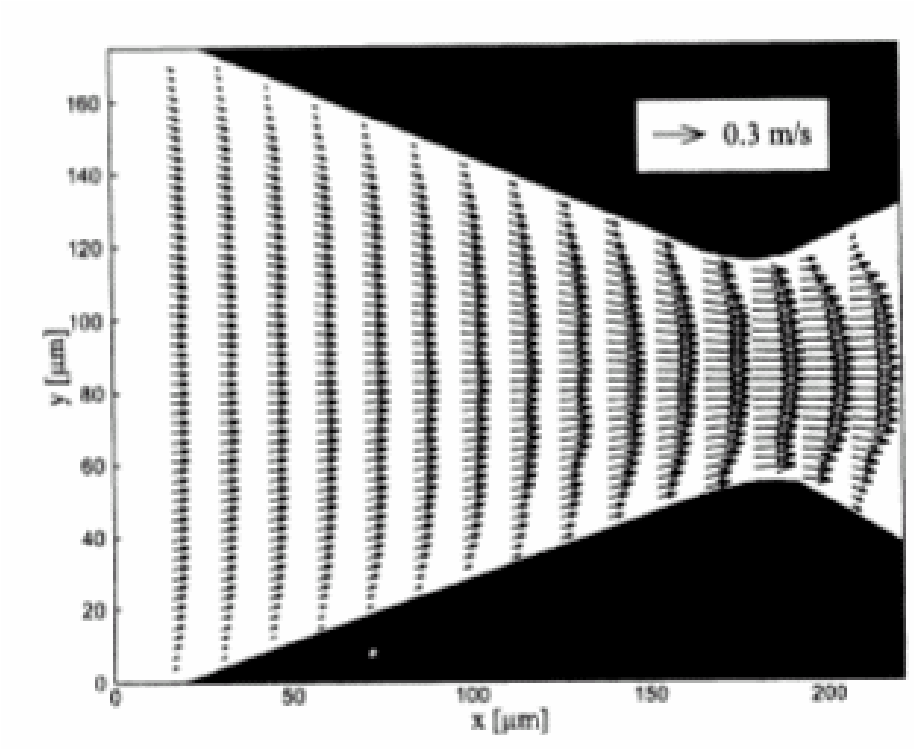


Results:

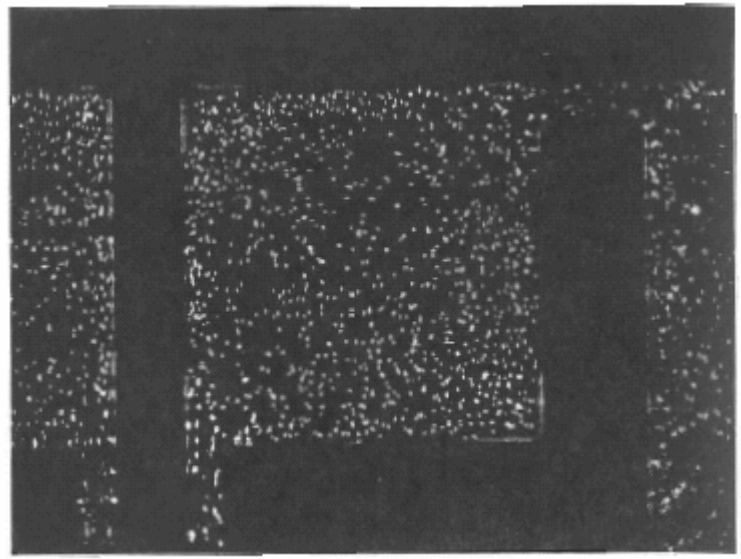
- Measured and analytical solution match perfectly
- possible to measure velocity profile down to within 450 nm of the wall
- position of the wall can be determined within 8 nm

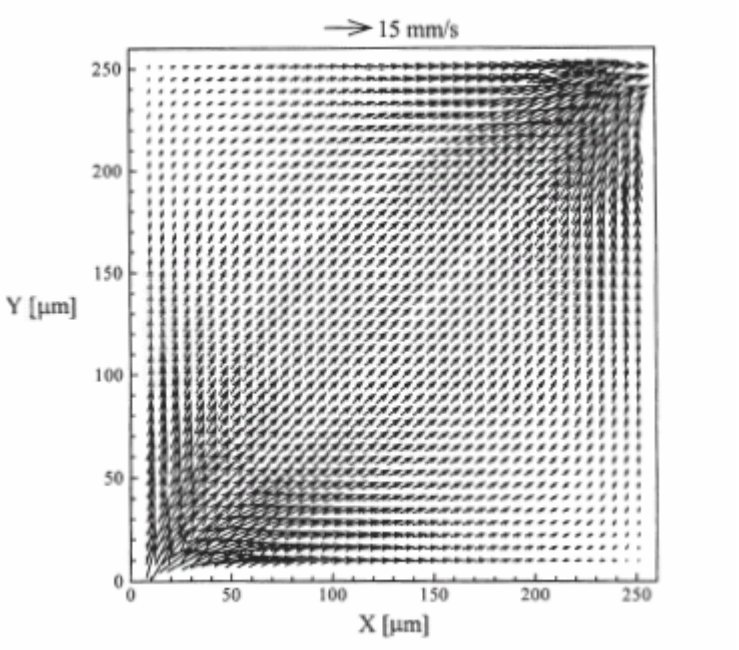


Flow in micronozzle



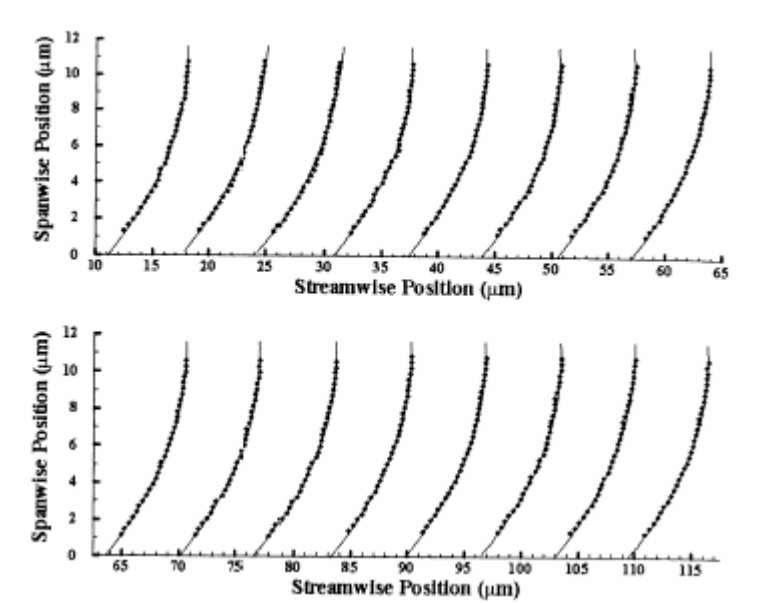
Flow in a biochip

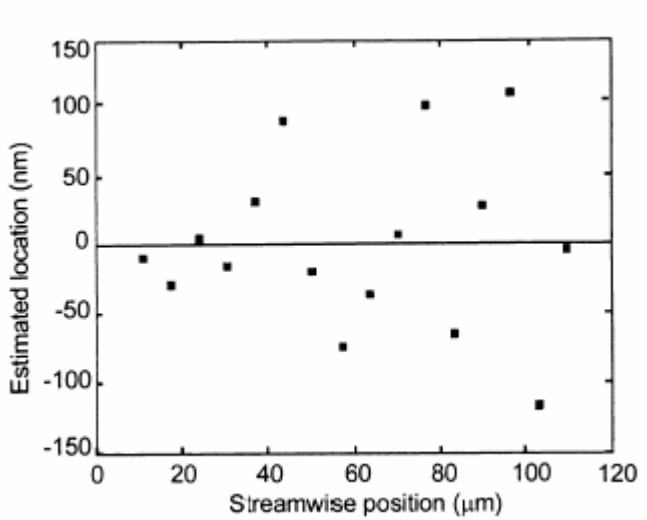




Extension of micro-PIV

- Microfluidic ”Nanoscope”
  by careful inspection of velocity profiles wall position can be obtained with high precision (<100nm)



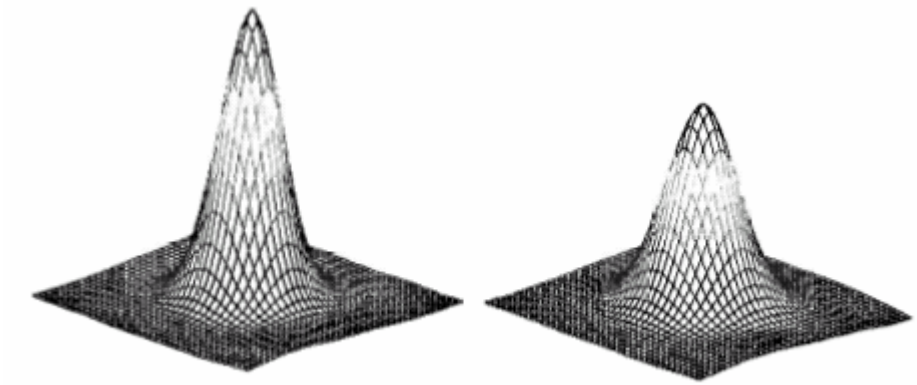


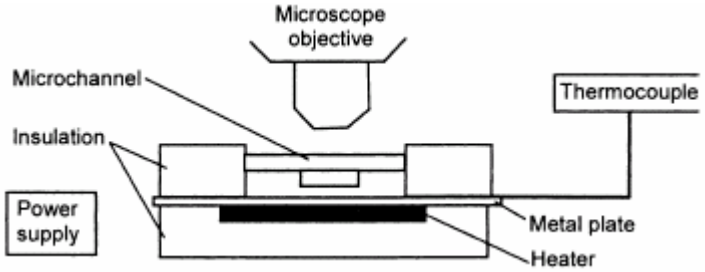
Useful to check channel geometry and hydrogel expansion

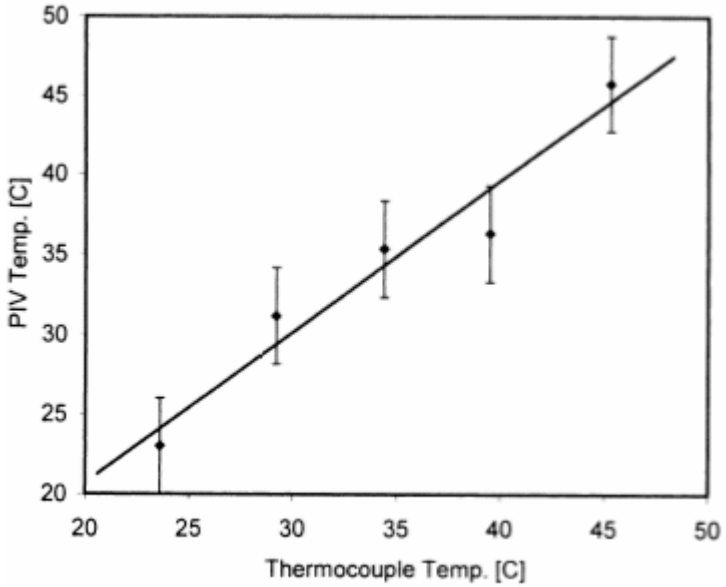
Micro-PIV thermometry

- Displacement (cross-correlation shift) and temperature (peak width) can be obtained independently

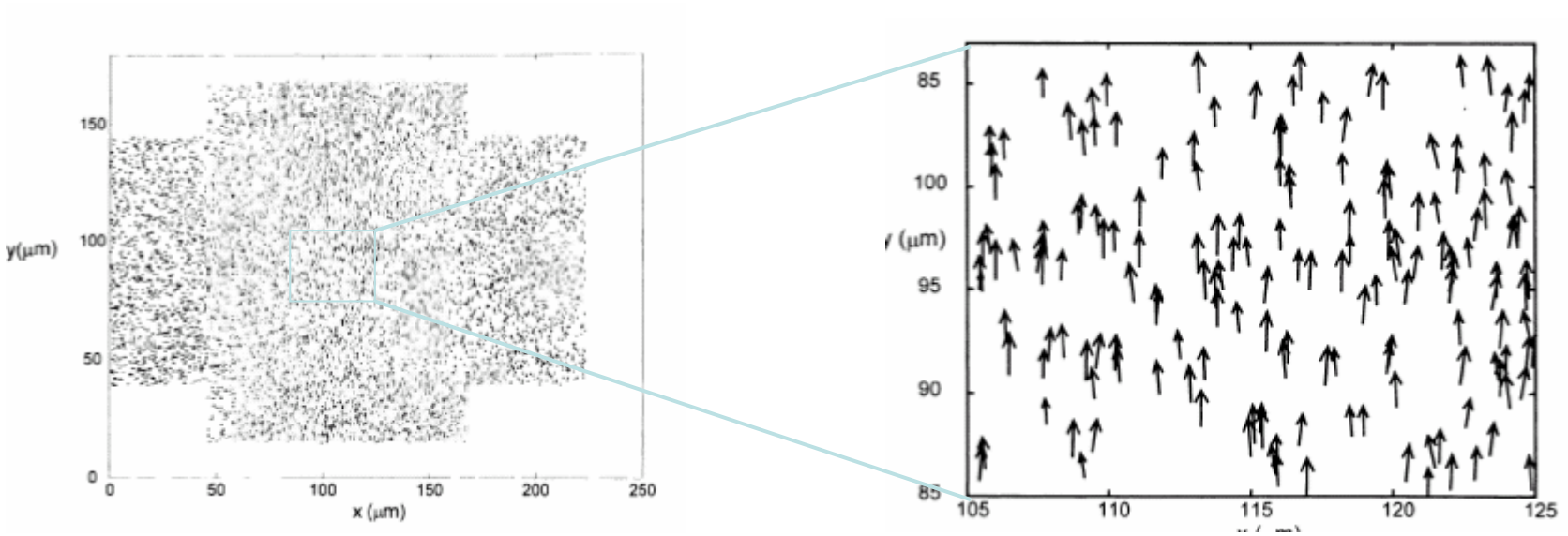
$$D = \frac{KT}{3\pi\mu d_p}$$







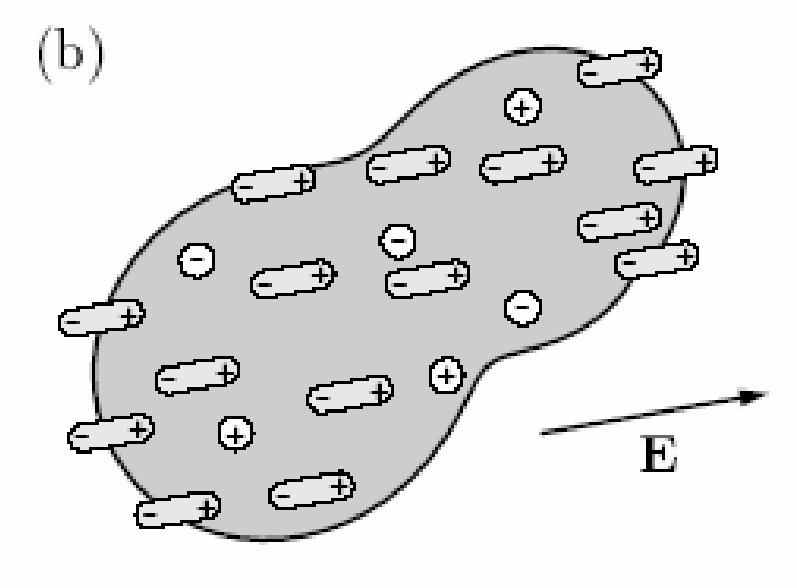
Particle tracking velocimetry or ”super-resolution” PIV



PIV resolution can be increased if after first PIV step the average velocity is used to track individual particles. In this case more than 5200 velocity measurements were performed using 10 image pairs.

Electric field in a continuous media: Polarization

- Non-constant electric field will cause polarization of the media:



$$F_i = \int_\Omega d\vec{r}\,\rho(\vec{r_0}+\vec{r})\left[E_i(\vec{r_0}) + r_j \partial_j E_i(\vec{r_0})\right] = QE_i(\vec{r_0}) + p_j \partial_j E_i(\vec{r_0})$$

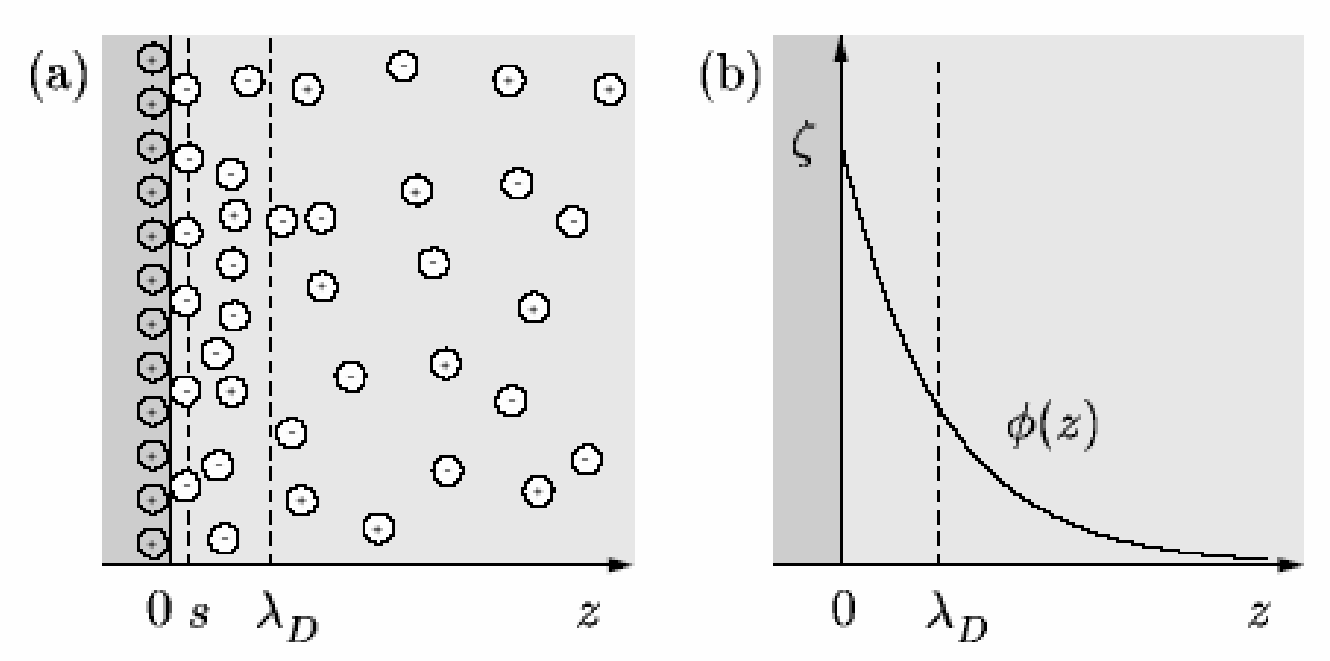
$$Q = \int_\Omega d\vec{r}\,\rho(\vec{r_0}+\vec{r})$$

$$p = \int_\Omega d\vec{r}\,\rho(\vec{r_0}+\vec{r})\vec{r}$$

Electrokinetic effects

- **Electrophoresis:** movement of charged surface (e.g. particle or molecule) relative to a stationary liquid.
- **Electroosmosis:** movement of liquid relative to a stationary charged surface (e.g. capillary tube)
- **Sedimentation potential**: electric potential created by moving charged particles
- **Streaming potential**: electric potential created by liquid moving relative to charged surface
- **Dielectrophoresis**: movement of uncharged with polarizability different from the liquid.

Debye layer in an electrolyte



- The potential at the Stern layer next to the surface is called zeta-potential ζ.

Electroosmotic flow

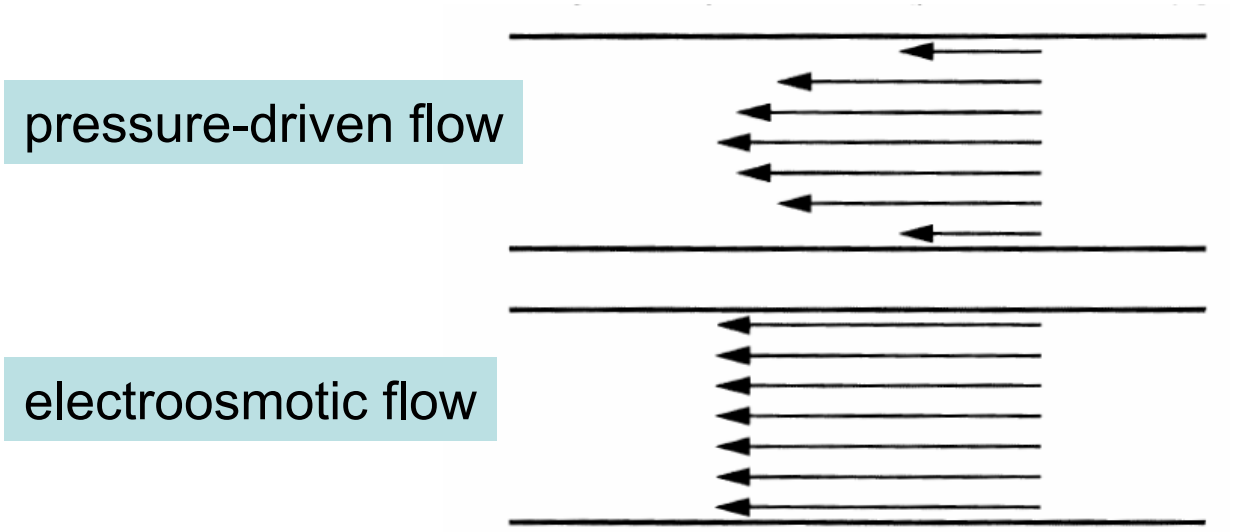
- Experimentally, if electric field is applied across a tube filled with electrolyte a uniform flow is induced:

dielectric constant

zeta potential

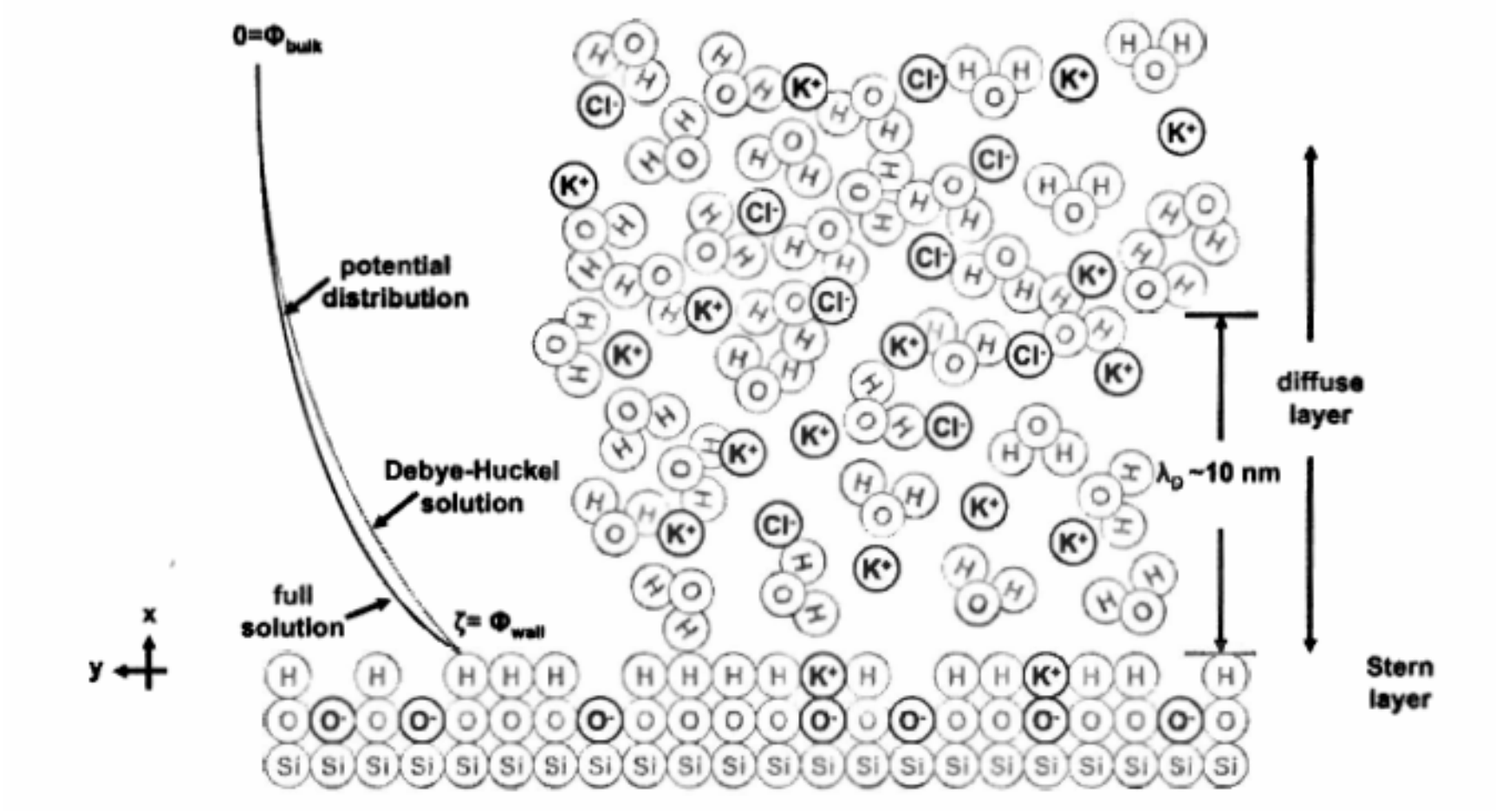
$$u = -\frac{\varepsilon\zeta}{\mu}E$$

viscosity



Origin of electroosmosis

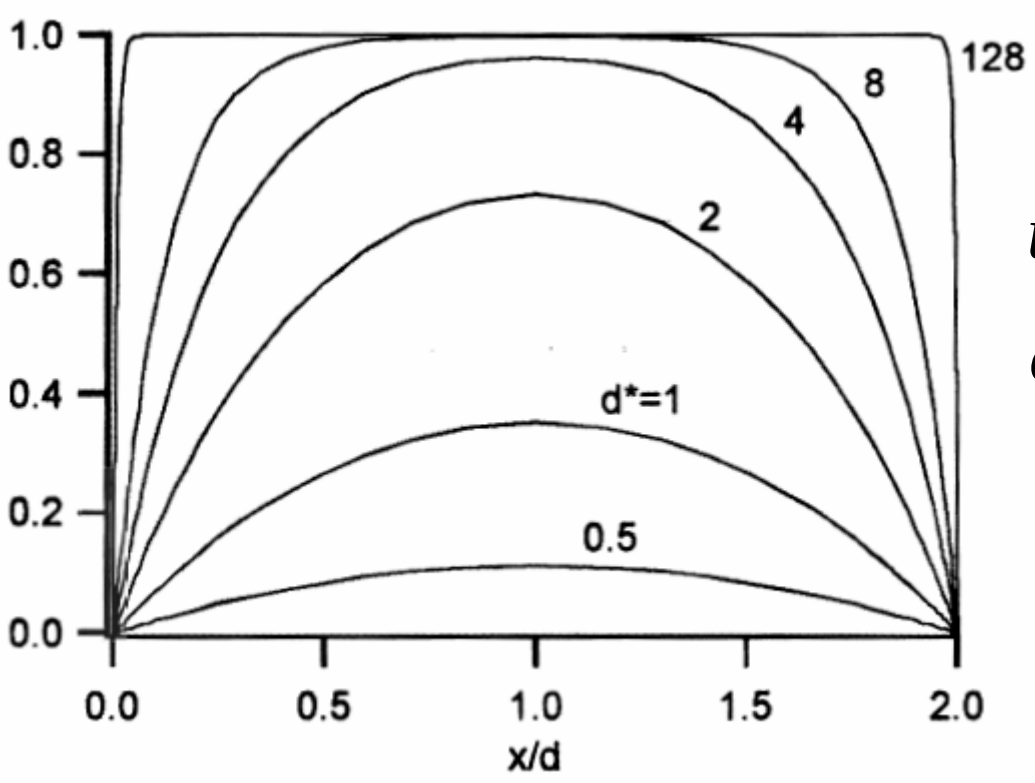
- Structure of the double layer



Debye-Huckel limit: $\phi = \zeta e^{-x/\lambda_D}$ $\quad u = -\dfrac{\varepsilon \zeta E_y}{\mu}\left(1 - e^{-x/\lambda_D}\right)$

Origin of electroosmosis

- Flow velocity profile between parallel plates



$$u^* = u\left(\varepsilon\zeta E / \mu\right)^{-1}$$

$$d^* = d / \lambda_D$$

Measuring of zeta potential

- tracking electroosmotic velocities
- streaming potential:
  - if a pressure driven flow is applied to an electrolyte, a potential will build up

$$\Delta V = \frac{\varepsilon \zeta}{\sigma \mu} \Delta p$$

Helmholtz-Smoluchowski equation

Electrophoretic effect

- Electrophoresis: motion of electrically charged molecule of particles in response to electric field
  Electrophoretic velocity

zeta potential at the particle surface

$$u = \frac{\varepsilon\zeta}{\mu}E \qquad \lambda_d \ll d \quad \text{(particle size)}$$

$$u = \frac{2}{3}\frac{\varepsilon\zeta}{\mu}E \qquad \lambda_d \gg d$$

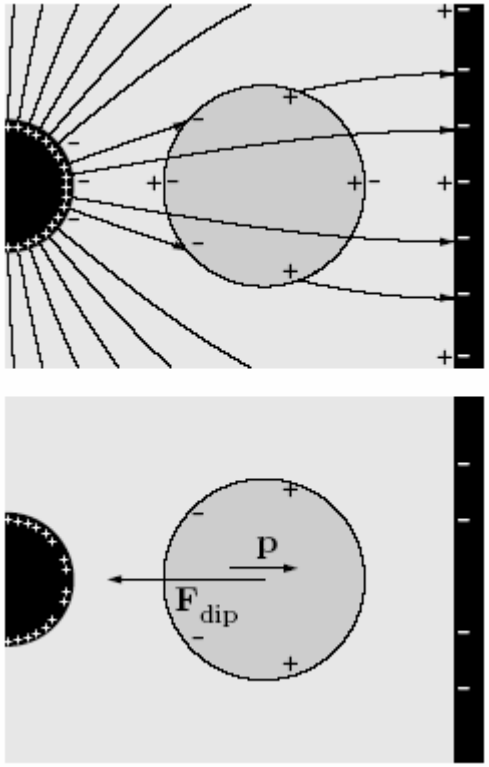
Generally: $\mu_{ep} = A\frac{\varepsilon\zeta}{\mu}E;\ \ \mu_{net,j} = \mu_{eo} + \mu_{ep,j}$

Resolution of electrophoretic system: $R_{mn} = \frac{\Delta t_{mn}}{w}$ ($\Delta t_{mn} \propto t$, $w \propto \sqrt{t}$)

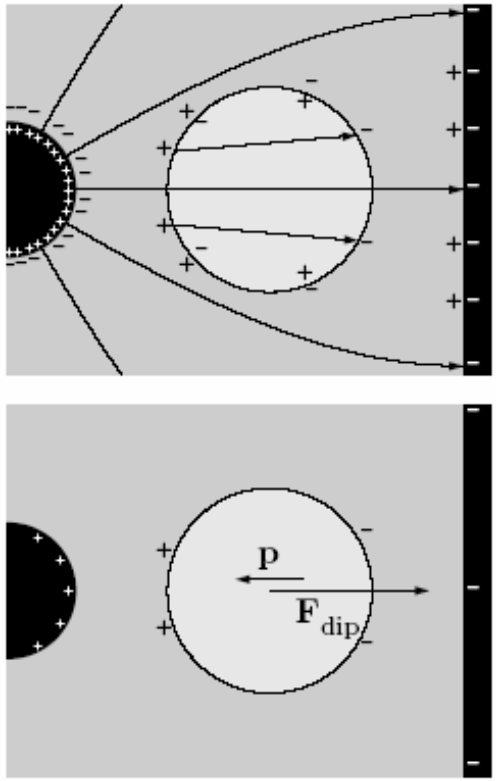
Dielectrophoresis

- **Dielectrophoresis:** movement of a charge neutral particle in a fluid induced by an inhomogeneous electric field (no DC field is necessary)

Particle is more polarizable than the fluid

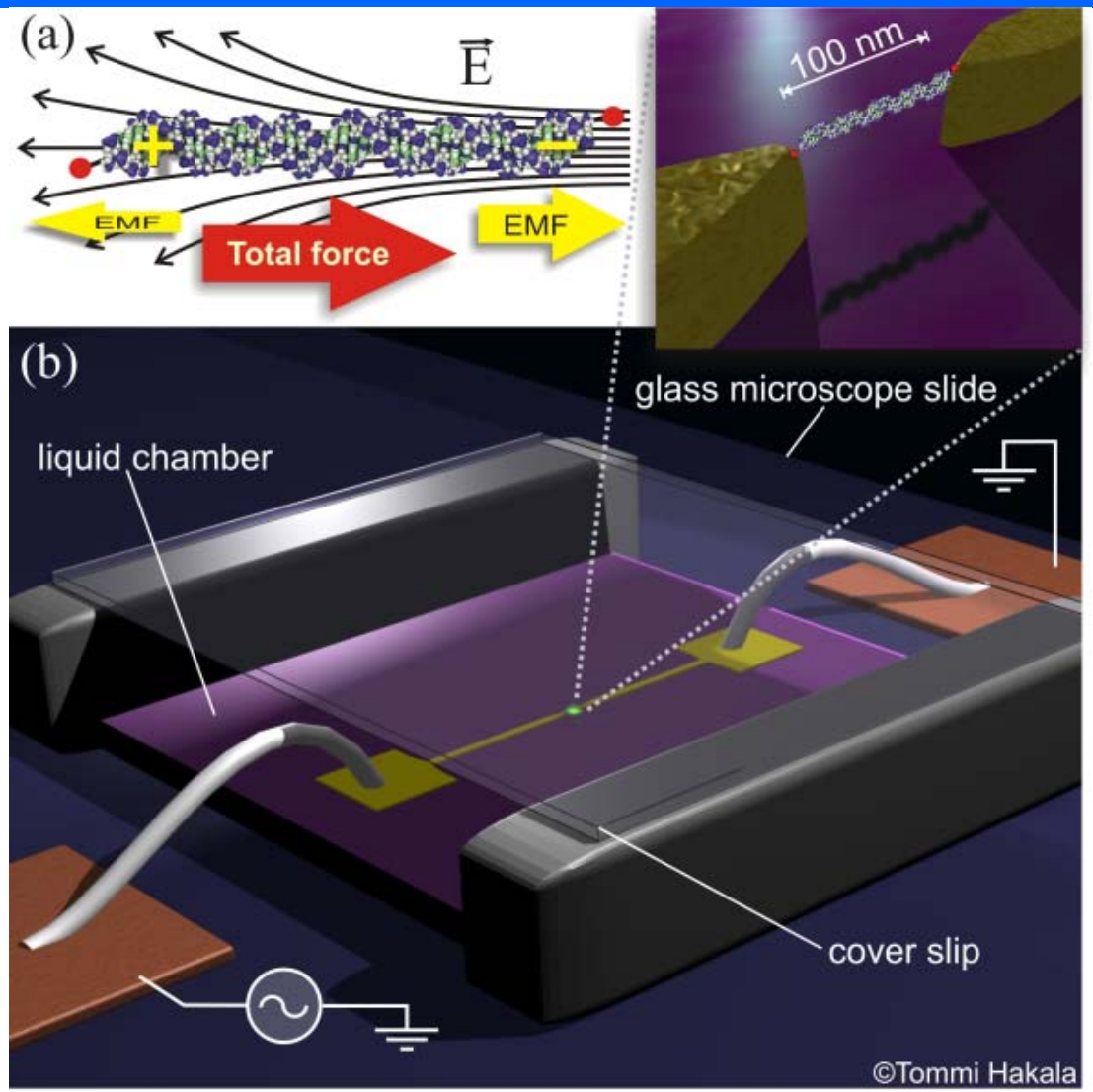


Particle is less polarizable than the fluid



Dielectrophoretic trapping of nanoparticles and molecules

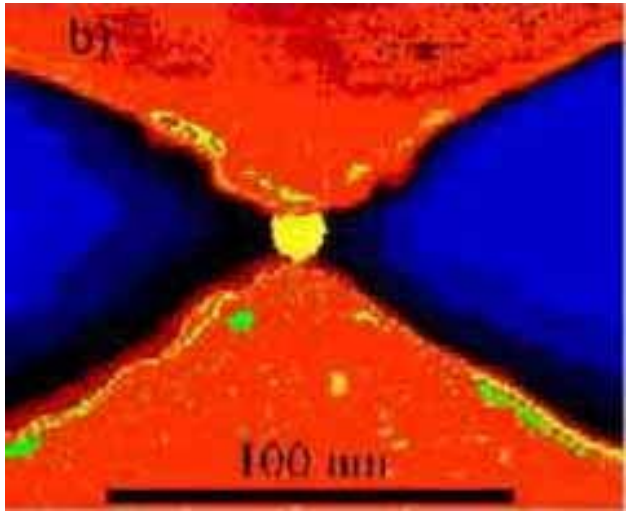
- strong gradient of electric field existing near a sharp electrode will attract small polarizable particles/molecules



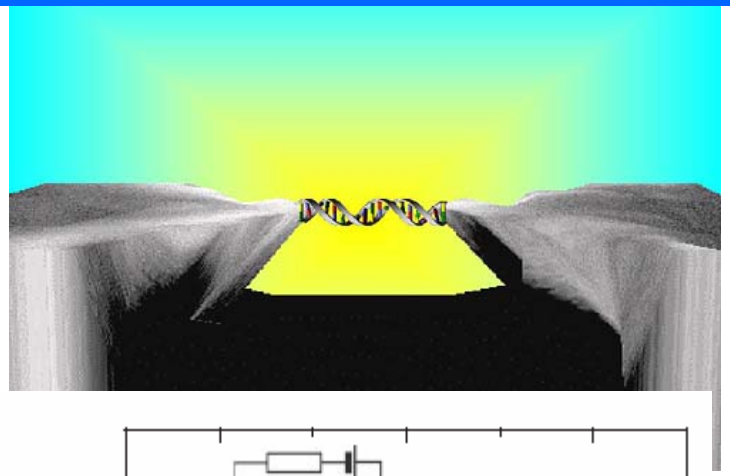
S. Tuukkanen, J.J. Toppari, A. Kuzyk, L. Hirviniemi, V.P. Hytönen, T. Ihalainen, and P. Törmä, Nano Lett. **6**, 1339 (2006)).

Dielectrophoretic trapping of nanoparticles and molecules

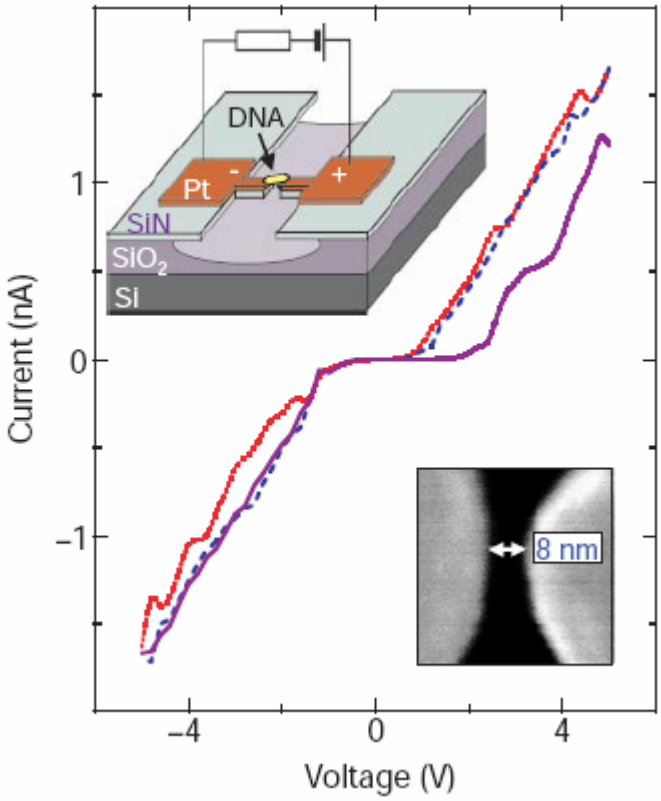
- Metal nanoparticle trapped between the electrodes



A. Bezryadin, C. Dekker, and G. Schmid, Appl. Phys. Lett. 71, 1273—1275 (1997).



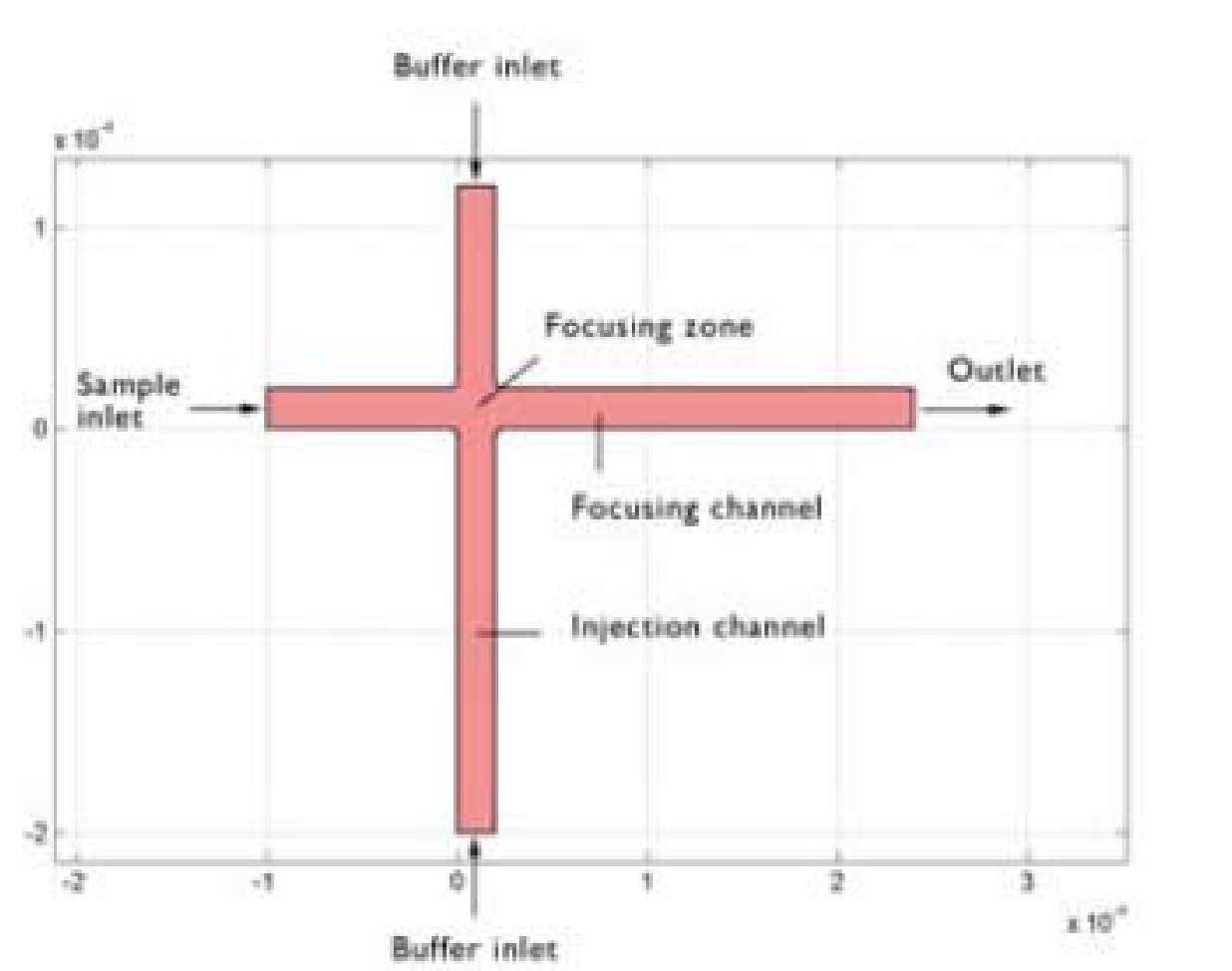
- Measurement of electrical characteristic of trapped polyC-polyG DNA



Danny Porath, Alexey Bezryadin, Simon de Vries & Cees Dekker, Nature 403, 635 (2000).

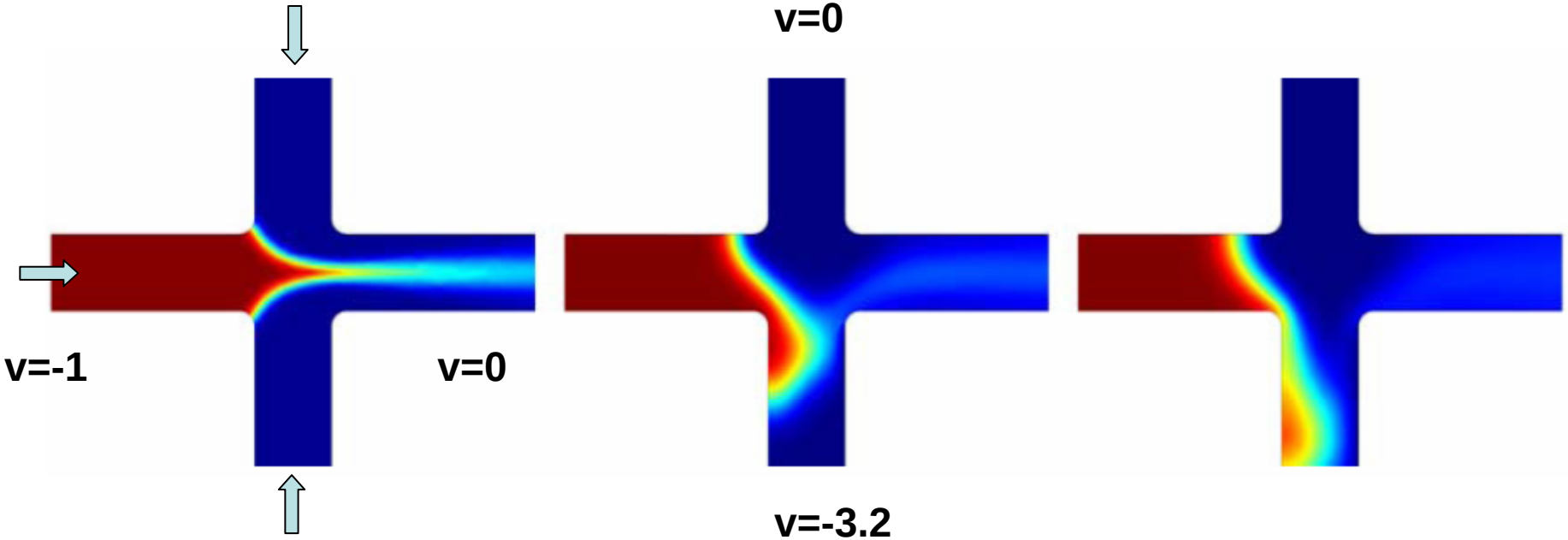
Electrokinetic valve

- Geometry



Electrokinetic valve

- Technique



Focusing

Injection