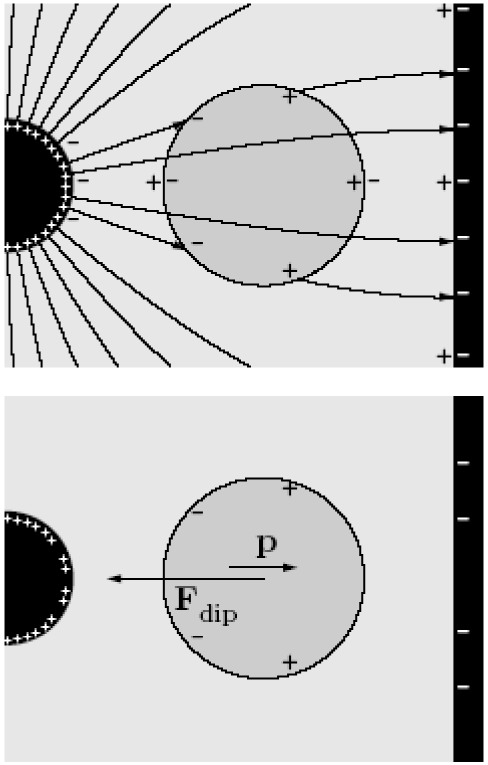
Lecture 9

Particle in an electrical field:
Electrophoresis, Dielectrophoresis
and Trapping

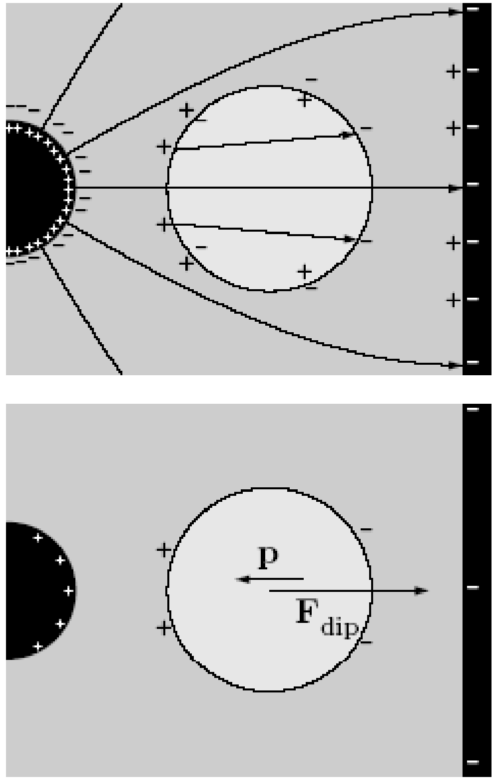
Dielectrophoresis

- **Dielectrophoresis:** movement of a charge neutral particle in a fluid induced by an inhomogeneous electric field (no DC field is necessary)

Particle is more polarizable than the fluid



Particle is less polarizable than the fluid



Electrical field in a media

- Assumptions:
  Electrostatic regime and continuous media

- Governing equations:
  - Maxwell:

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{D} = \nabla \cdot \left(\varepsilon \vec{E} \right) = \rho_{el}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\vec{J} = \sigma_{el} \vec{E}$$

  - As electrical field is irrotational

$$\vec{E} = -\nabla \phi$$

$$\nabla^2 \phi(r) = -\frac{1}{\varepsilon} \rho_{el}$$

Poisson equation

Force caused by electric field

- Consider a particle located at r_0 in an electric field:

$$\vec{F}_{el} = \int_{\Omega} \rho_{el} \vec{E} dr$$

$$F_i^{el} = \int_{\Omega} \rho_{el}(r_0 + r) E_i(r_0 + r) dr = \int_{\Omega} \rho_{el}(r_0 + r) \left[E_i(r_0) + r_i \left.\frac{\partial E_i}{\partial r_i}\right|_{r=r_0} \right] dr =$$

$$= Q E_i(r_0) + p_i \left.\frac{\partial E_i}{\partial r_i}\right|_{r=r_0}$$

$$Q = \int_{\Omega} \rho_{el}(r_0 + r)\, dr$$

$$\vec{p} = \int_{\Omega} \rho_{el}(r_0 + r) \vec{r}\, dr$$

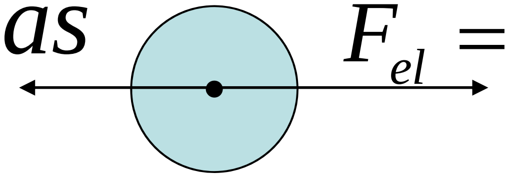
$$\vec{F} = Q\vec{E}(r_0) + \vec{p} \cdot \nabla \vec{E}$$

tensor gradient of electric field

Electrophoresis

- Let’s consider a charged spherical particle in a low conductivity liquid.

$$F_{drag} = 6\pi\eta a\vec{s}$$



$$\vec{F}_{el} = Ze \cdot \vec{E}$$

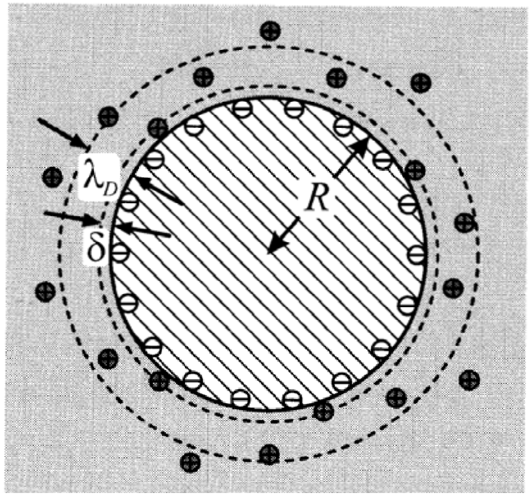
- the drift velocity is:

$$u = \frac{Ze \cdot E}{6\pi\eta a}$$

- the equation is valid also for ions (except $H^+$and OH^-) assuming hydration radius of approx. 0.2nm.

Electrophoretic effect in electrolyte

- In an electrolyte, the Debye-layer screening should be taken into account
- Movement of a particle can be approximated as:



$$v = v_i + v_o = \frac{QE}{6\pi\eta R}\left(1 + \lambda_D / R\right)^{-1}$$

zeta potential at the particle surface

$$u = \frac{\varepsilon\varsigma}{\eta}E \qquad \lambda_d \ll d \quad \text{(particle size)}$$

$$u = \frac{2}{3}\frac{\varepsilon\varsigma}{\eta}E \qquad \lambda_d \gg d$$

Generally: $\mu_{ep} = A\frac{\varepsilon\varsigma}{\eta}E; \;\; \mu_{net,j} = \mu_{eo} + \mu_{ep,j}$

Resolution of electrophoretic system: $R_{mn} = \frac{\Delta t_{mn}}{w}$

$\boxed{\propto t}$

$\boxed{\propto \sqrt{t}}$

Dielectrophoresis

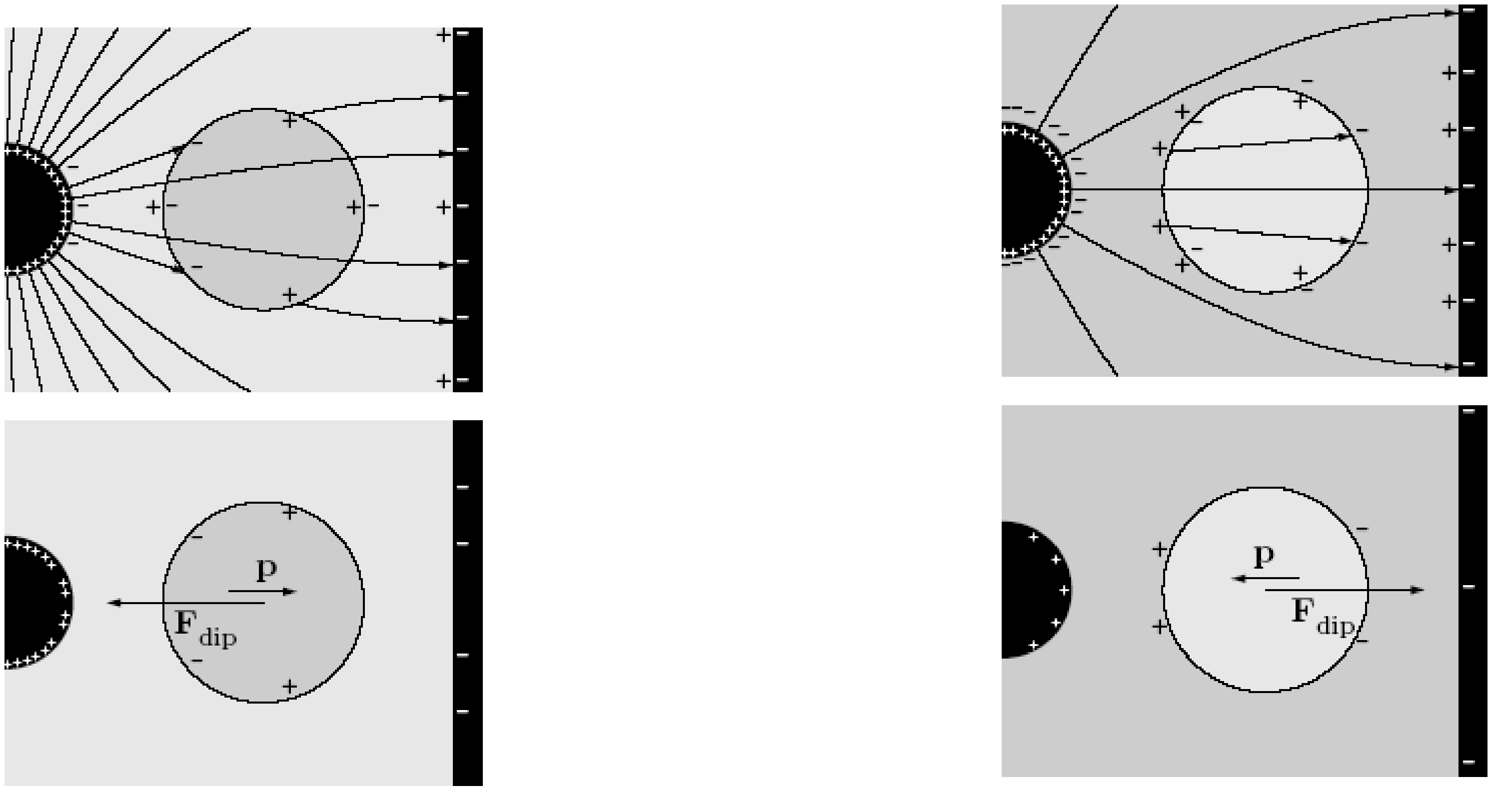
- **Dielectrophoresis:** movement of a charge neutral particle in a fluid induced by an inhomogeneous electric field (no DC field is necessary)

induced dipole moment

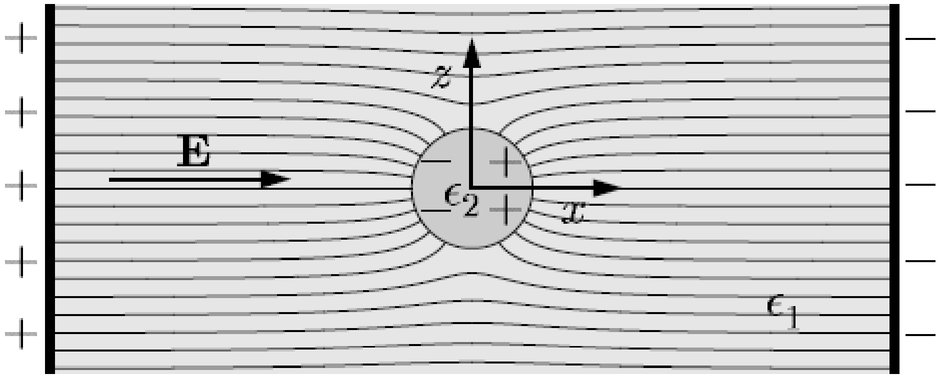
$$\vec{F}_{dip} = \vec{p} \cdot \nabla \vec{E}; \quad \vec{p} = \alpha \vec{E}$$

Particle is more polarizable than the fluid

Particle is less polarizable than the fluid



Dielectrophoretic trapping



- Dipole moment:

$$\mathbf{p} = 4\pi\epsilon_1 \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1}\, a^3 \mathbf{E}_0.$$

$$K(\epsilon_1, \epsilon_2) \equiv \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1}.$$

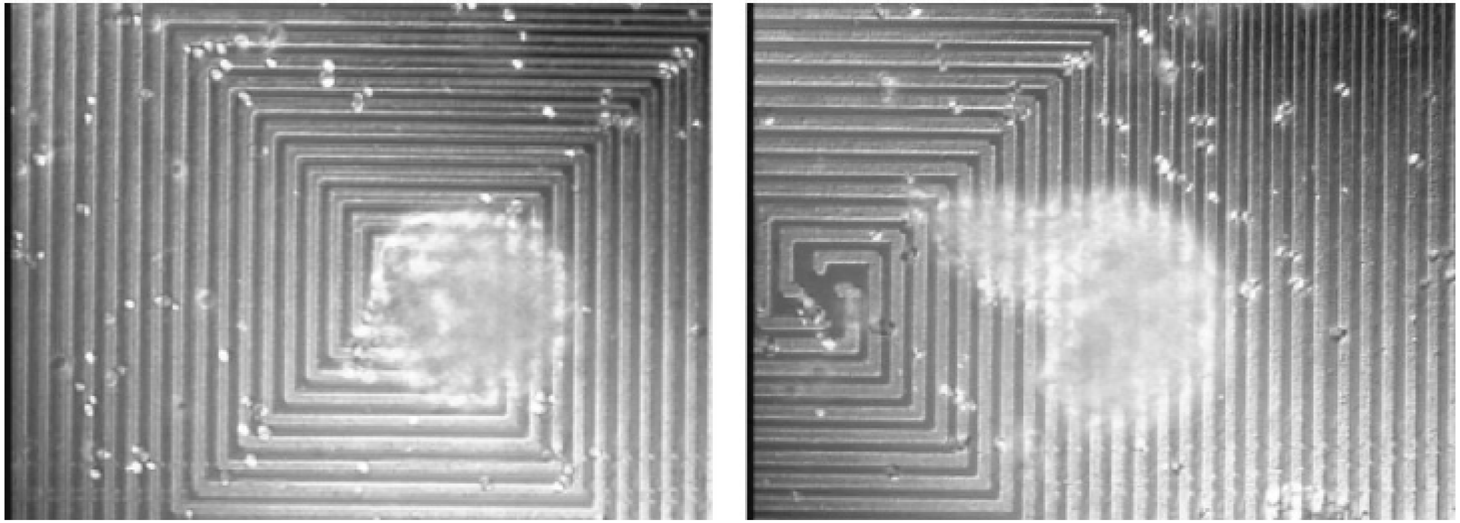
- Dielectrophoretic force on a sphere

$$\mathbf{F}_{\mathrm{DEP}}(\mathbf{r}_0, t) = 2\pi\epsilon_1 \frac{\epsilon_2(\omega) - \epsilon_1(\omega)}{\epsilon_2(\omega) + 2\epsilon_1(\omega)}\, a^3 \nabla\left[\mathbf{E}(\mathbf{r}_0, t)^2\right].$$

Dielectrophoretic trapping

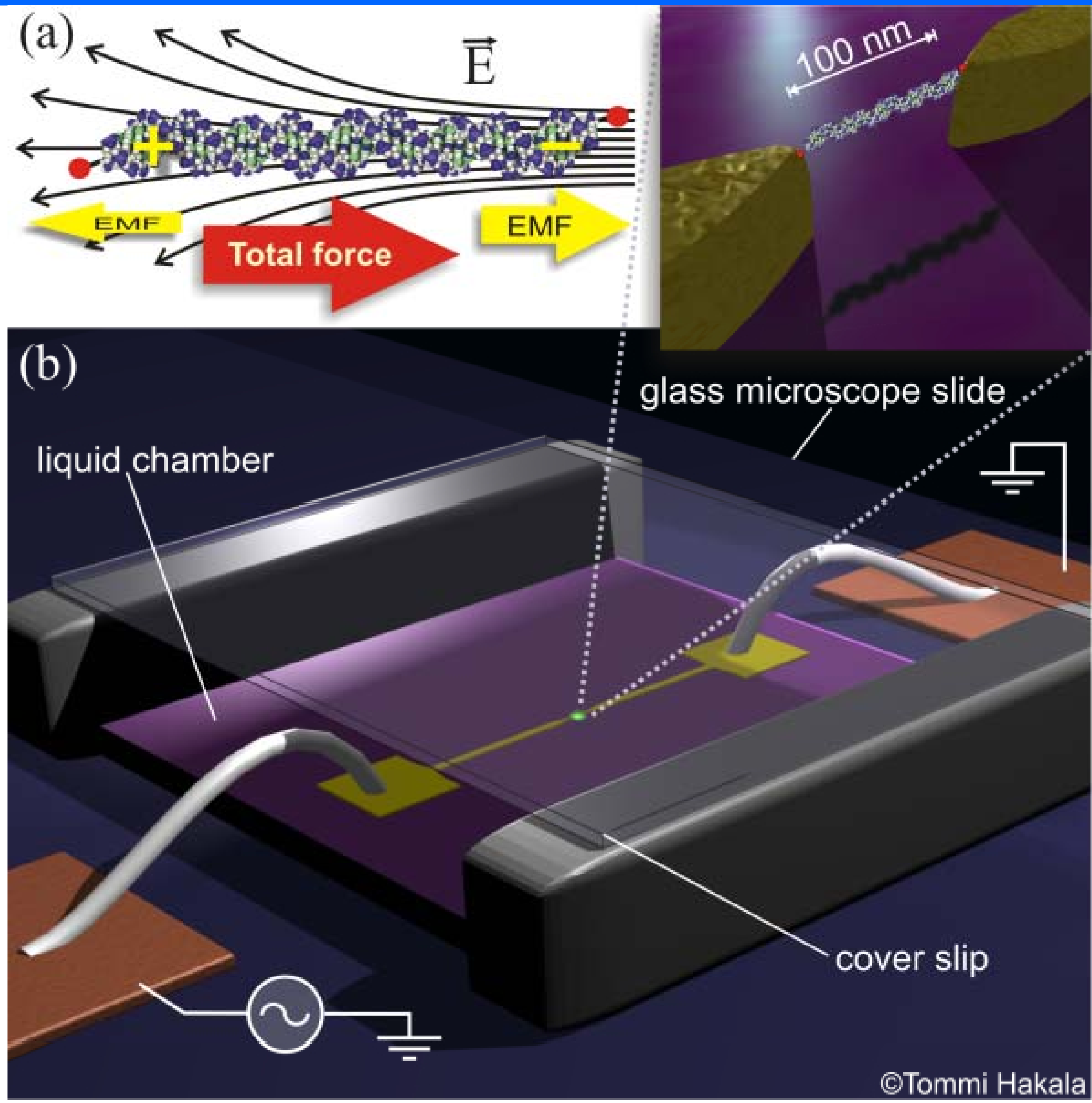
- DEP trap for bacterial cells

(a) (b)



Dielectrophoretic trapping of nanoparticles and molecules

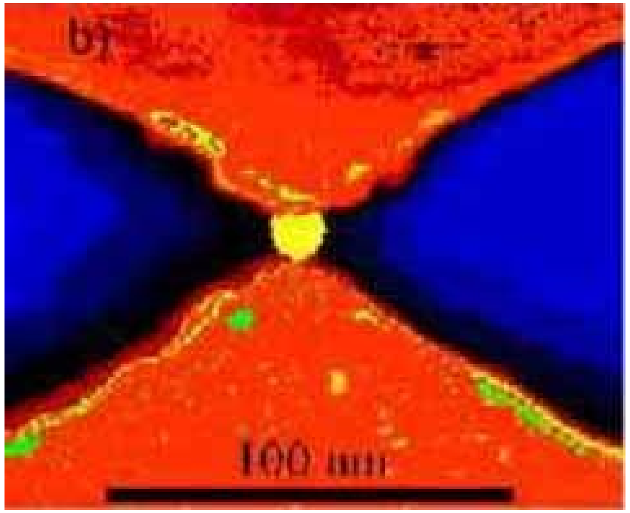
- strong gradient of electric field existing near a sharp electrode will attract small polarizable particles/molecules



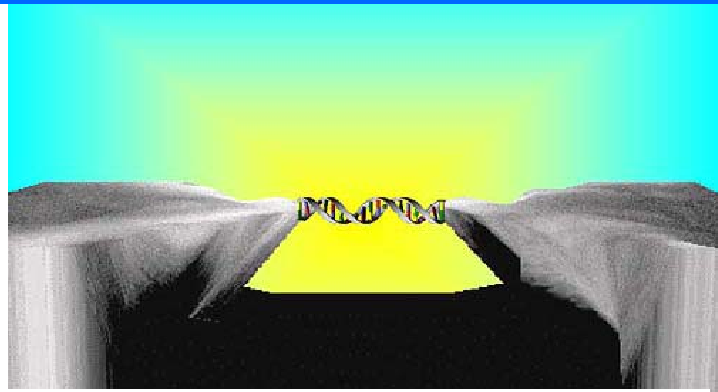
S. Tuukkanen, J.J. Toppari, A. Kuzyk, L. Hirviniemi, V.P. Hytönen, T. Ihalainen, and P. Törmä, Nano Lett. **6**, 1339 (2006)).

Dielectrophoretic trapping of nanoparticles and molecules

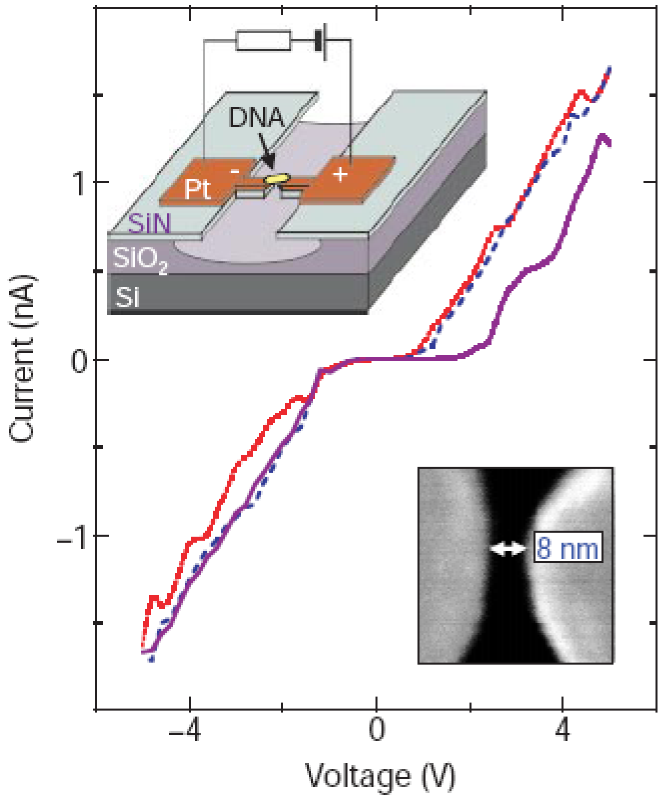
- Metal nanoparticle trapped between the electrodes



A. Bezryadin, C. Dekker, and G. Schmid, Appl. Phys. Lett. 71, 1273—1275 (1997).



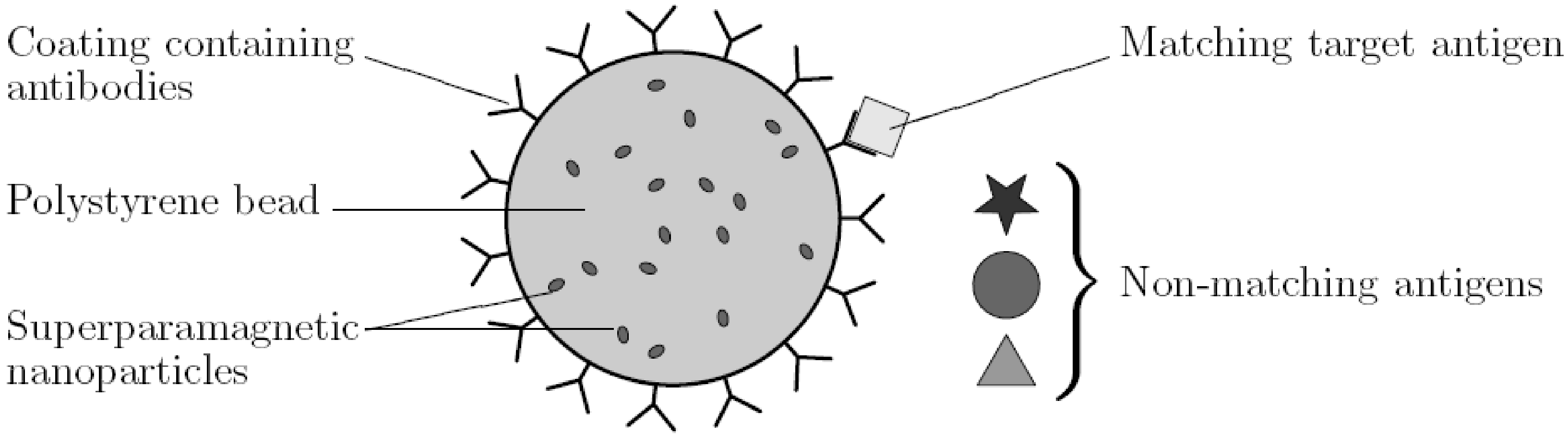
- Measurement of electrical characteristic of trapped polyC-polyG DNA



Danny Porath, Alexey Bezryadin, Simon de Vries & Cees Dekker, Nature 403, 635 (2000).

Magnetophoresis

- **Magnetophoresis** – magnetic analogue of dielectrophoresis. Most of biomaterials are non-magnetic and require labelling (actually, advantage!)



Superparamagnetic particles: small ~ 10nm particles with no hysteresis and no momentum in zero field, but large susceptibility in magnetic field

Magnetophoresis

- Magnetostatic Maxwell equations

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 J_{ext} + \mu_0 J_{mag}$$

current density bound to magnetic material

- Magnetic moment

$$\vec{m} = I_{mag} A\vec{n}$$

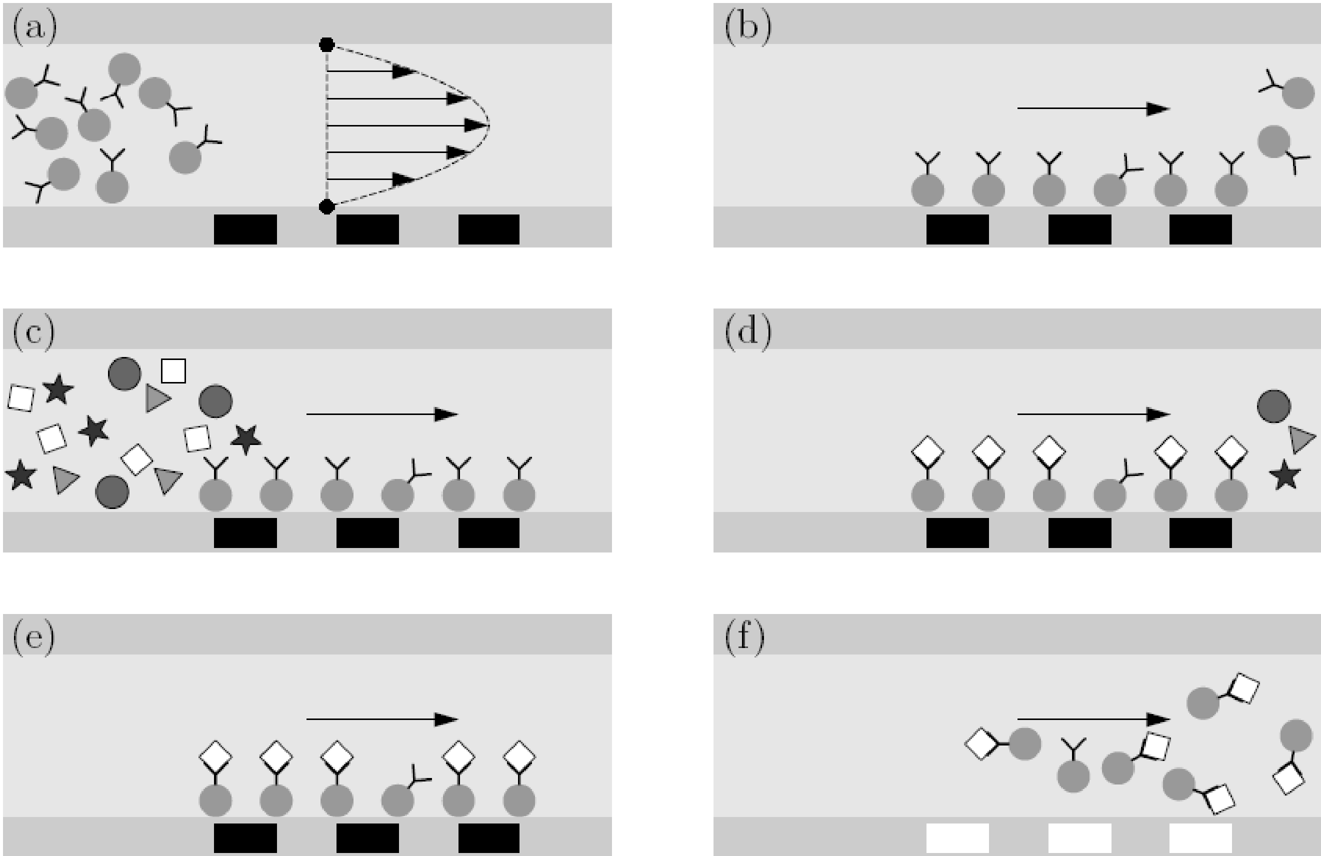
- Magnetization is defined as magnetic moment density

$$\mathbf{M}(\mathbf{r}_0) = \lim_{\mathrm{Vol}(\Omega^*)\to 0} \left[\frac{1}{\mathrm{Vol}(\Omega^*)} \int_{\Omega^*} \mathrm{d}\mathbf{r}\, \mathbf{m}(\mathbf{r}_0 + \mathbf{r}) \right]$$

$$\vec{J}_{mag} = \nabla \times \vec{M}$$

Magnetophoresis

- Principle of magnetic separation



Magnetophoresis

- Magenetic induction

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$$

$$\nabla \times \vec{H} = J_{ext}$$

$$\chi \equiv \left(\frac{\partial M}{\partial H}\right)_{V,T}$$

magnetic susceptibility

$$M = \chi H$$

$$B = \mu_0(H + M) = \mu_0(1+\chi)H \equiv \mu_0\mu_r H \equiv \mu H$$

relative permeability

permeability

Basic equations for magnetophoresis

- The differential of the free energy density

$$d\,\mathrm{F} = -\vec{B} \cdot d\vec{H}$$

- The magnetostatic force

$$dF = -\int_{body} \vec{B} \cdot \nabla \vec{H} = \mu_0 \int_{body} \vec{M} \cdot \nabla \vec{H}_{ext}$$

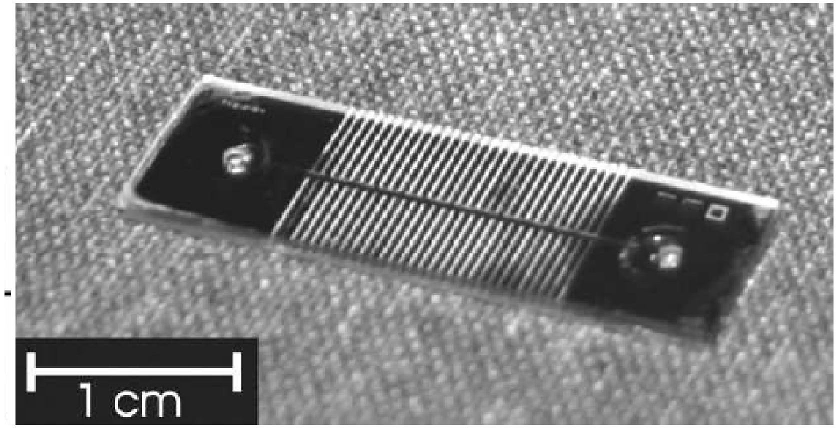
- By analogy to the dielectrophoresis:

$$\vec{M} = 3 \frac{\mu - \mu_0}{\mu + 2\mu_0} \vec{H}_{ext}$$

$$F_{MAP} = 2\pi\mu_0 K(\mu_0, \mu) a^3 \nabla \left[H_{ext}(r_0)^2 \right]$$

Calculating magnetic bead motion

- The recipe:
  - Find the magnetophoretic force on a particle,
  - take it equal to the viscous drag and then
  - find velocity and calculate trajectory



$$dr = udt = \left[v(r) + \frac{2\pi\mu_0}{6\pi\mu a} K(\mu_0, \mu) a^3 \nabla \left[H_{ext}(r_0)^2 \right] \right] dt$$

COMSOL example: Magnetic drug targeting

- Model